Departement Physik, Universität Basel
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Mechanik, Herbstsemester 2026

Blatt 9

Abgabe: 17.11.2026, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
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(1) **Hamiltonians for well-known problems** (5 Punkte)
Starting from the Lagrangian L, calculate the Hamiltonian H and write down Hamilton's equations of motion.

(a) One-dimensional harmonic oscillator

(b) Three-dimensional harmonic oscillator

(c) Charged particle in a magnetic field

(d) Particle in a central potential.
Hint: $H = H(r, \phi, p_r, p_\phi)$ where $p_r = \dfrac{\partial L}{\partial \dot{r}}$ and $p_\phi = \dfrac{\partial L}{\partial \dot{\phi}}$ are the canonical momenta associated with the (standard) polar coordinates r and ϕ.

(e) Double pendulum (see Blatt 7 problem 1(a) for L).
Result for H:

$$\begin{aligned} H(\phi_1, \phi_2, p_{\phi_1}, p_{\phi_2}) = &- ga(m_1 + m_2)\cos(\phi_1) - gam_2\cos(\phi_2) \\ &+ \frac{m_2 p_{\phi_1}^2 + (m_1 + m_2) p_{\phi_2}^2 - 2m_2 p_{\phi_1} p_{\phi_2}\cos(\phi_1 - \phi_2)}{a^2 m_2 \left(2m_1 + m_2 - m_2\cos(2(\phi_1 - \phi_2))\right)} \,. \end{aligned}$$

Hint: Write L as

$$L(\boldsymbol{\phi}, \dot{\boldsymbol{\phi}}) = \frac{1}{2}\dot{\boldsymbol{\phi}}^{\mathrm{T}} A \dot{\boldsymbol{\phi}} - V(\boldsymbol{\phi}),$$

where A is a symmetric positive-definite matrix.

(2) **Canonical transformations** (5 Punkte)
We consider a system with one degree of freedom ($f = 1$) and want to study canonical transformations from (q, p) to (Q, P) generated by $F_2(q, P, t)$ which leads to

$$Q = \frac{\partial F_2}{\partial P} \qquad\qquad p = \frac{\partial F_2}{\partial q} \,.$$

Construct suitable generators F_2 for the following cases:

(a) Identical transformation: $Q = q$, $P = p$.

(b) Galilei transformation, for which Q, P belong to a system moving with velocity v with respect to the original system: $q = Q + vt$, $p = P + mv$. Write down the new Hamiltonian $K(Q, P)$ that is related to the original Hamiltonian H by the relation $K = H + \frac{\partial F_2}{\partial t}$.

(c) A general point transformation from q to the new coordinate Q, i.e., $Q = g(q, t)$. Construct a generator F_2 such that the new momentum P is *proportional* to the original momentum p. Show that the transformation rule for the momentum can also be obtained from the fact that the Lagrangian is invariant under a point transformation: $L(q, \dot{q}; t) = \tilde{L}(Q, \dot{Q}; t)$.