

Mechanik, Herbstsemester 2026

Blatt 2

Abgabe: 29.09.2026, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**

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Schriftlicher Test: Montag, 14. Dezember 2026, 13.15 - 15 Uhr

Hilfsmittel: Zwei handbeschriebene Seiten A4

Mündliche Vorlesungsprüfung (Examen): Montag/Dienstag 1./2. Februar 2027

(1) **Particle in a circular paraboloid** (3 Punkte)

A particle with mass m slides on the inner surface of a circular paraboloid described by the equation $z = a(x^2 + y^2)$ and is subject to a (homogeneous) gravitational force acting in the negative z -direction.

- (a) Describe the problem in cylindrical coordinates (r, φ, z) . Write down the equations of motion (they contain the constraint forces) for the two independent coordinates r and φ .
- (b) Eliminate the constraint forces and find the equations of motion for r and φ . Result:

$$2\dot{r}\dot{\varphi} + r\ddot{\varphi} = 0 \quad (1)$$

$$\ddot{r}(1 + 4r^2a^2) + 4r\dot{r}^2a^2 - r\dot{\varphi}^2 + 2gra = 0. \quad (2)$$

- (c) Now use the Lagrange formalism to determine the equations of motion, Eqs. (1) and (2).

You are not asked to solve the equations of motion.

(2) **Numerical treatment of a particle in a circular cone** (3 Punkte + 1 Bonusp.)

In the lecture we derived the equations of motion of a particle moving on the inner surface of a circular cone (opening angle $\alpha = \pi/4$). In cylindrical coordinates (r, φ, z) , they read

$$2\dot{r}\dot{\varphi} + r\ddot{\varphi} = 0 \quad (3)$$

$$2\ddot{r} - r\dot{\varphi}^2 + g = 0. \quad (4)$$

- (a) Show that (3) leads to $r^2\dot{\varphi} = \text{const.}$; we'll call this constant l_z/m for reasons that will become obvious soon.
- (b) Use (a) to eliminate $\dot{\varphi}$ from (4) and solve the remaining second-order equation for r numerically by writing a Julia notebook (or other means).
Hint: you can find a template for a program (cone_skeleton) on adam.

- (c) The naive numerical solution will give you $r(t)$. How can you easily modify (b) to obtain $\varphi(t)$?
 Plot trajectories of r as a function of φ . Describe and discuss the trajectories for different initial conditions. Find initial conditions that lead to circular orbits.
- (d) Check that your solution conserves the energy $E = m\dot{r}^2 + l_z^2/(2mr^2) + mgr$.

Particularly nice graphics will get a bonus point.

- (3) **Is the Lagrangian unique?** (2 Punkte)
 Let $L(\mathbf{q}, \dot{\mathbf{q}}, t)$ be a Lagrangian of a system. The equations of motion of the system are then given by Lagrange's equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0.$$

Show that

$$\tilde{L}(\mathbf{q}, \dot{\mathbf{q}}, t) = L(\mathbf{q}, \dot{\mathbf{q}}, t) + \frac{d}{dt} G(\mathbf{q}, t)$$

also fulfills Lagrange's equations.

- (4) **Charge in an electromagnetic field** (2 Punkte)
 The electromagnetic force on a particle with charge q is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

Show that $\mathbf{F}$ can be obtained from a generalized (velocity-dependent) potential $U(\mathbf{r}, \mathbf{v}) = q(\phi(\mathbf{r}) - \mathbf{v} \cdot \mathbf{A}(\mathbf{r}))$ by using

$$\mathbf{F} = -\nabla U + \frac{d}{dt} \frac{\partial U}{\partial \mathbf{v}}.$$

Hint: The electrostatic potential ϕ and the vector potential $\mathbf{A}$ are connected to the electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ by $\mathbf{E} = -\nabla\phi - \frac{\partial \mathbf{A}}{\partial t}$ und $\mathbf{B} = \nabla \times \mathbf{A}$.