

Mechanik, Herbstsemester 2026

Blatt 1

Abgabe: 22.09.2026, 12:00H **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**

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Die **Übungskreditpunkte** erhält, wer sowohl 50% der Punkte aus den Hausaufgaben erreicht als auch 50% der Punkte aus dem schriftlichen Test am Ende des Semesters.

- (1) **How quickly can a mass slide from r_A to r_B ?** (6 Punkte)

We consider a point mass that slides without friction on a curve $y(x)$ in the xy-plane connecting the two points $r_A = (0, 0)$ and $r_B = (2, -1)$. The mass starts at r_A with velocity 0 and is subject to the Earth's gravitational field that is assumed to be homogeneous and point in the negative y-direction.

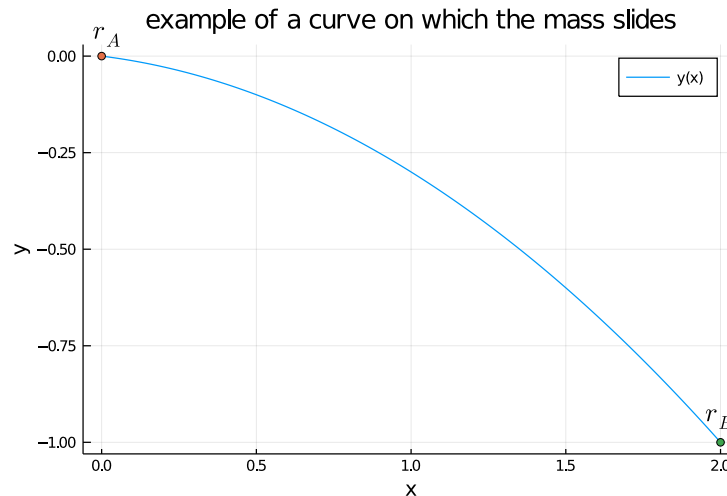


Figure 1:

- (a) Use energy conservation to calculate the velocity of the particle at a given y -coordinate. Result: $v = \sqrt{2g(-y)}$.
Show that the total time that the particle needs to reach r_B can be expressed as
- $$T = \int_{r_{Ax}}^{r_{Bx}} dx \frac{\sqrt{1+y'^2}}{\sqrt{2g(-y)}}.$$
- (b) Calculate the time T exactly if $y(x)$ is a straight line. Result: $T_{\text{straight}} = \sqrt{10/9.81}\text{s}$.
- (c) Write a computer program (using Julia or some other programming language) to calculate T for an arbitrary curve $y(x)$. Confirm that you obtain the result of (b) in the case of a straight line. Now try modifications of a straight line and explore curves for which $T < T_{\text{straight}}$. What is the minimal time that you can find??

(2) **Weak and strong form of Newton's 3rd law** (4 Punkte)

Consider a system of particles $\mathbf{r}_i$ with masses m_i , $i = 1, \dots, N$. Using (the weak form of) Newton's 3rd law, $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$, where $\mathbf{F}_{ij}$ is the force of the i th particle on the j th particle, we proved that the total momentum $\mathbf{P} = \sum_i m_i \frac{d\mathbf{r}_i}{dt}$ is conserved.

However, Newton's 3rd law is not valid for magnetic forces, as we will now show.

Imagine a point charge q_1 moving along the x -axis towards the origin with velocity $\mathbf{v}$. Its magnetic field $\mathbf{B}$ is *not* given by the Biot-Savart law (which only holds for steady currents), but still circles around the axis in a manner suggested by the right-hand rule, see Fig. 2. Now suppose this charge encounters an identical one, q_2 , proceeding at the same speed along the y -axis towards the origin.

- (a) Which direction has the magnetic force $\mathbf{F}_{12}$ on q_2 due to the first particle q_1 ?
Which direction has the magnetic force $\mathbf{F}_{21}$ on q_1 due to the second particle q_2 ?
No calculation required; use geometric arguments.
- (b) Conclude that Newton's 3rd law is violated. Does this mean momentum conservation does not hold in electrodynamics?
- (c) The strong form of Newton's 3rd law requires in addition to $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ that $\mathbf{F}_{ij}$ is parallel to the line $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ joining particles j and i and is used to prove the conservation of angular momentum.
Now consider two charged particles moving in parallel, but not perpendicular to $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$. Show that $\mathbf{F}_{12} = -\mathbf{F}_{21}$, but $\mathbf{F}_{12}$ is not parallel to $\mathbf{r}_{12}$, i.e., this situation violates the strong form of Newton's 3rd law.

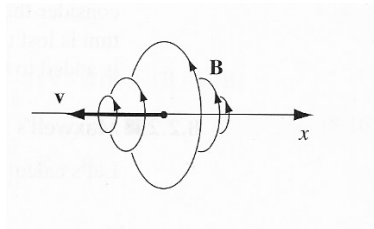


Figure 2: Magnetic field of a moving charge (as you will learn in the lecture Electrodynamics. Check “Liénard-Wiechert potentials” if you are curious).