

Quantenmechanik, Herbstsemester 2025

Blatt 0

Präsenzübung am 17. September 2025

(1) **Spin 1/2**

Consider a particle with spin $\frac{1}{2}$. Before each of the measurements discussed below, the particle has been prepared in the state

$$|\psi\rangle = i\frac{1}{\sqrt{5}}|\uparrow\rangle_z + \frac{2}{\sqrt{5}}|\downarrow\rangle_z.$$

Here, $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ are the eigenstates of the operator $\hat{S}_z$ with eigenvalues $\frac{\hbar}{2}$, $-\frac{\hbar}{2}$, respectively.

NB: In the z -basis, we have $\hat{S}_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{S}_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, and $\hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

- (a) What is the probability to obtain the value $\frac{\hbar}{2}$ when measuring $\hat{S}_z$?
- (b) Compute the expectation value of $\hat{S}_z$.
- (c) Compute the expectation value of $\hat{S}_x$ and $\hat{S}_y$.
- (d) Rewrite the vector $|\uparrow\rangle_x$, which is the eigenstate of $\hat{S}_x$ with eigenvalue $\frac{\hbar}{2}$, as a linear combination of $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$. Same for $|\downarrow\rangle_x$.
- (e) What is the probability to obtain the value $\pm\frac{\hbar}{2}$ when $\hat{S}_x$ is measured? Check that your result is consistent with (c).
- (f) Repeat (d) and (e) for $\hat{S}_y$.
- (g) What is the probability to obtain the value 0 when measuring $\hat{S}_x$?