Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 207 36 92, Christoph.Bruder@unibas.ch)

Mechanik, Herbstsemester 2024

Blatt 9

Abgabe: 19.11.2024, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
Tutor: Julian Arnold Zi.: 4.10; julian.arnold@unibas.ch

(1) **Symmetric top in a uniform gravitational field ("heavy top")** (4 Punkte)
In the lecture we discussed the motion of a symmetric top placed in a uniform gravitational field and fixed in one point by expressing the Lagrangian using Euler angles,

$$L(\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}) = \frac{1}{2} I_1 (\dot{\phi}^2 \sin^2\theta + \dot{\theta}^2) + \frac{1}{2} I_3 (\dot{\phi}\cos\theta + \dot{\psi})^2 - mg\ell\cos\theta \,.$$

(a) Assume that the top rotates around the vertical axis of the lab system, $\theta = 0$. Show that in this case, $L_z = L_3$ where $L_z = p_\phi = \partial L/\partial\dot{\phi}$ is the angular momentum about the vertical axis and $L_3 = p_\psi = \partial L/\partial\dot{\psi}$ the angular moment about the body axis x_3.

(b) Plot the effective potential for a representative choice of parameters. Find the condition under which the vertical rotation is *stable* under small perturbations θ . Hint: expand the effective potential around $\theta = 0$.

(c) Bonus points: solve the equations of motion (for general θ) numerically. Confirm the nutation behavior discussed in the lecture and the stability condition discussed in (b).

(2) **Hamiltonians for well-known problems** (6 Punkte)
Starting from the Lagrangian L, calculate the Hamiltonian H and write down Hamilton's equations of motion.

(a) One-dimensional harmonic oscillator

(b) Three-dimensional harmonic oscillator

(c) Charged particle in a magnetic field

(d) Particle in a central potential.
Hint: $H = H(r, \phi, p_r, p_\phi)$ where $p_r = \dfrac{\partial L}{\partial \dot{r}}$ and $p_\phi = \dfrac{\partial L}{\partial \dot{\phi}}$ are the canonical momenta associated with the (standard) polar coordinates r and ϕ.

(e) Double pendulum (see Blatt 7 problem 1(a) for L).
Result for H:

$$\begin{aligned} H(\phi_1, \phi_2, p_{\phi_1}, p_{\phi_2}) = &- ga(m_1+m_2)\cos(\phi_1) - gam_2\cos(\phi_2) \\ &+ \frac{m_2 p_{\phi_1}^2 + (m_1+m_2)p_{\phi_2}^2 - 2m_2 p_{\phi_1} p_{\phi_2}\cos(\phi_1-\phi_2)}{a^2 m_2\left(2m_1 + m_2 - m_2\cos(2(\phi_1-\phi_2))\right)} \,. \end{aligned}$$

Hint: Write L as

$$L(\boldsymbol{\phi}, \dot{\boldsymbol{\phi}}) = \frac{1}{2}\dot{\boldsymbol{\phi}}^{\mathrm{T}} A \dot{\boldsymbol{\phi}} - V(\boldsymbol{\phi}),$$

where A is a symmetric positive-definite matrix.