Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 207 36 92, Christoph.Bruder@unibas.ch)

Mechanik, Herbstsemester 2024

Blatt 6

Abgabe: 29.10.2024, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
Tutor: Tobias Kehrer Zi.: 4.48; tobias.kehrer@unibas.ch

(1) **Scattering angle and differential cross section** (4 Punkte)
A particle is scattered from a scattering center with potential $V(r) = \frac{\alpha}{r^2}$ where $\alpha > 0$.

(a) Calculate the minimal distance r_{min} from the scattering center for a given impact parameter s and energy E.

(b) Calculate the scattering angle θ as a function of s and E.

(c) Calculate the differential cross section $\frac{d\sigma}{d\Omega}(\theta)$.

Discuss your results and compare with the case of Rutherford scattering.

(2) **Coriolis force** (2 Punkte)
At a polar angle θ, a projectile is fired eastward with speed v_0 at an angle α above the ground. Show that the southward (in the northern hemisphere) and eastward deflections due to the Coriolis force are (to first order in ω)

$$d_{\text{south}} = (4\omega v_0^3/g^2)\cos\theta\cos\alpha\sin^2\alpha \, ,$$
$$d_{\text{east}} = (4\omega v_0^3/g^2)\sin\theta(\cos^2\alpha\sin\alpha - (1/3)\sin^3\alpha) \, .$$

Hint: The first term in d_{east} arises because the flight time is modified due to the vertical component of the Coriolis force.

(3) **Driven damped harmonic oscillator** (4 Punkte)
Consider a damped harmonic oscillator subject to the external driving force $mf(t) = mf_0\cos(\omega t)$. Thus, the equation of motion reads

$$\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = f(t)$$

where $\gamma > 0$ is the damping constant.

(a) Assume $f_0 = 0$ and discuss the dependence of the solution on γ for the initial conditions $x(0) = a$, $\dot{x}(0) = 0$.

(b) Show that for $f_0 \neq 0$ there is a solution of the form

$$x(t) = Re\{\chi(\omega) f_0 \exp(i\omega t)\} = A(\omega)\cos(\omega t + \varphi) \, .$$

Determine the complex quantity $\chi(\omega)$ and discuss its real and imaginary parts as a function of the (real) frequency ω. How are $\text{Re}\chi$ and $\text{Im}\chi$ related to the amplitude A and phase φ of the oscillation? Sketch A and φ as a function of ω for different values of the ratio γ/ω_0. What happens for $\gamma \to 0$?

(c) Bonus points: Now regard ω as a *complex* quantity and investigate the structure of $\chi(\omega)$ in the complex plane. What happens for $\gamma \to 0$?

(4) **Bonus problem: Restricted three-body problem** (5 Bonuspunkte)
The restricted three-body problem consists of two masses in circular orbits about each other and a third body of much smaller mass whose effect on the two larger bodies can be neglected.

(a) Define an effective potential $V(x, y)$ for this problem by going to a rotating frame in which the x-axis is the line of the two larger masses. Sketch the function $V(x, 0)$ and show that there are two *valleys* (points of stable equilibrium) corresponding to the two masses. Also show that there are three *hills* (three points of unstable equilibrium).

(b) Use a computer to calculate some orbits for the restricted three-body problem. Many orbits will end with ejection of the small mass. Start by assuming a position and a vector velocity for the small mass.

See Goldstein, Chapter 3.12 for some basic facts about the three-body problem and https://en.wikipedia.org/wiki/Three-body_problem for interesting comments and references to recent developments.

Also, there is a popular Chinese science-fiction novel that features the three-body problem.