Departement Physik, Universität Basel
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Mechanik, Herbstsemester 2024

Blatt 3

Abgabe: 8.10.2024, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
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(1) **Principle of least action for a particle under a constant force** (3 Punkte)
A particle is subjected to the potential $V(x) = -Fx$ where F is constant. The particle travels from $x = x_0 = 0$ to $x = x_1$ in a time t_1. Use the ansatz $x(t) = A + Bt + Ct^2$ and find the values of A, B, and C by minimizing the action explicitly such that the action is a minimum.

(2) **Minimal surface** (4 Punkte)
Consider a surface defined by rotating the curve $r = f(z)$ about the z-axis; we further assume $z \in [-L/2, L/2]$ and $f(-L/2) = f(L/2) = R$.

(a) Express the area of the surface S as an integral. Result: $S = 2\pi \int_{-L/2}^{L/2} f\sqrt{1 + f'^2}\mathrm{d}z$.

(b) Finde the function f that minimizes the area.
Hint: Use the “first integral” of the Euler-Lagrange equation discussed in the lecture.
Hint: $\int \frac{\mathrm{d}x}{\sqrt{x^2-1}} = \mathrm{acosh}(x)$
Remark: a soap film clamped to two circles of radius R separated by a distance L will assume this shape.

(c) Discuss your solution carefully. Is it compatible with all values of R/L? Can you find a minimal surface for the case in which the solution obtained (b) is not possible?

(3) **Bead on a stick** (3 Punkte)
(continuation of the example discussed in the lecture)
A stick is pivoted at the origin and is arranged to swing around in the xy-plane at constant angular velocity ω. A bead of mass m slides frictionless along the stick. Let r be the radial position of the bead. Show that the quantity

$$E = \sum_{i=1}^{f} \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

is conserved. Explain why this is *not* the energy of the bead.

(4) **Minimum or saddle** (3 Bonuspunkte)
Consider a one-dimensional harmonic oscillator that has the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2 \,.$$

Let $x_0(t)$ be a function that yields a stationary value of the action. Then we know that $x_0(t)$ fulfills the Euler-Lagrange equation, $m\ddot{x}_0 = -m\omega^2 x_0$. Consider a variation on this path, $x_0(t) + \eta(t)$, where $\eta(t_1) = \eta(t_2) = 0$.

(a) Calculate the action $S[x_0 + \eta]$ for $t_1 = 0$, $t_2 = \tau$.

(b) Show that it is always possible to find a function η such that $S[x_0 + \eta] - S[x_0]$ is positive. Conclude that $S[x_0]$ can never be a maximum of the action.

(c) Find a combination of τ and $\eta(t)$ such that $S[x_0+\eta]-S[x_0]$ is negative. Conclusion?