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Mechanik, Herbstsemester 2024

Blatt 1

Abgabe: 24.9.2024, 12:00H
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Die **Übungskreditpunkte** erhält, wer sowohl 50% der Punkte aus den Hausaufgaben erreicht als auch 50% der Punkte aus dem schriftlichen Test am Ende des Semesters.

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(1) **How quickly can a mass slide from r_A to r_B?** (6 Punkte)
We consider a point mass that slides without friction on a curve $y(x)$ in the xy-plane connecting the two points $r_A = (0,0)$ and $r_B = (2,-1)$. The mass starts at r_A with velocity 0 and is subject to the Earth's gravitational field that is assumed to be homogeneous and point in the negative y-direction.

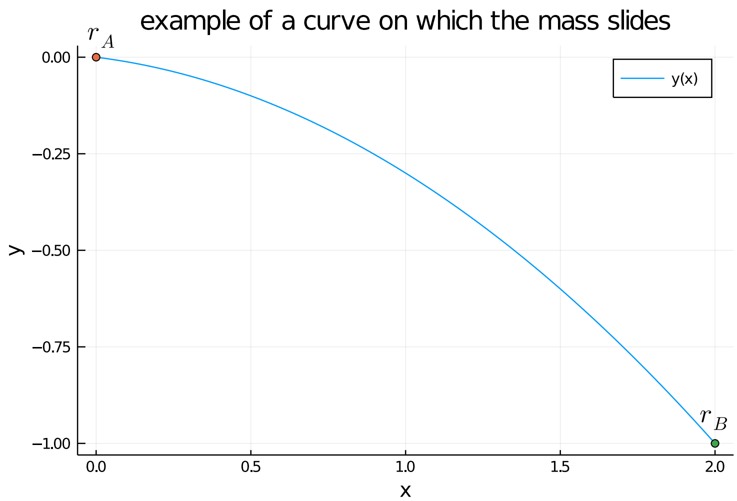

Figure 1:

(a) Use energy conservation to calculate the velocity of the particle at a given y-coordinate. Result: $v = \sqrt{2g(-y)}$.
Show that the total time that the particle needs to reach r_B can be expressed as $T = \int_{r_{Ax}}^{r_{Bx}} \mathrm{d}x \frac{\sqrt{1+y'^2}}{\sqrt{2g(-y)}}$.

(b) Calculate the time T exactly if $y(x)$ is a straight line. Result: $T_{\text{straight}} = \sqrt{10/9.81}\text{s}$.

(c) Write a computer program (using Julia or some other programming language) to calculate T for an arbitrary curve $y(x)$. Confirm that you obtain the result of (b) in the case of a straight line. Now try modifications of a straight line and explore curves for which $T < T_{\text{straight}}$. What is the minimal time that you can find??

(2) **Weak and strong form of Newton's 3rd law** (4 Punkte)

Consider a system of particles $\mathbf{r}_i$ with masses m_i, $i = 1, \ldots, N$. Using (the weak form of) Newton's 3rd law, $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$, where $\mathbf{F}_{ij}$ is the force of the ith particle on the jth particle, we proved that the total momentum $\mathbf{P} = \sum_i m_i \frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t}$ is conserved.

However, Newton's 3rd law is not valid for magnetic forces, as we will now show.

Imagine a point charge q_1 moving along the x-axis towards the origin with velocity $\mathbf{v}$. Its magnetic field $\mathbf{B}$ is *not* given by the Biot-Savart law (which only holds for steady currents), but still circles around the axis in a manner suggested by the right-hand rule, see Fig. 2. Now suppose this charge encounters an identical one, q_2, proceeding at the same speed along the y-axis towards the origin.

(a) Which direction has the magnetic force $\mathbf{F}_{12}$ on q_2 due to the first particle q_1? Which direction has the magnetic force $\mathbf{F}_{21}$ on q_1 due to the second particle q_2? No calculation required; use geometric arguments.

(b) Conclude that Newton's 3rd law is violated. Does this mean momentum conservation does not hold in electrodynamics?

(c) The strong form of Newton's 3rd law requires in addition to $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ that $\mathbf{F}_{ij}$ is parallel to the line $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ joining particles j and i and is used to prove the conservation of angular momentum.
Now consider two charged particles moving in parallel, but not perpendicular to $\mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2$. Show that $\mathbf{F}_{12} = -\mathbf{F}_{21}$, but $\mathbf{F}_{12}$ is not parallel to $\mathbf{r}_{12}$, i.e., this situation violates the strong form of Newton's 3rd law.

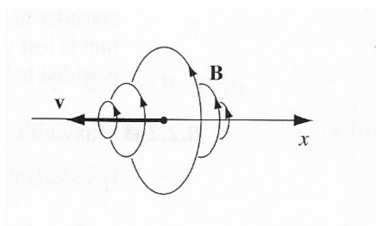

Figure 2: Magnetic field of a moving charge (as you will learn in the lecture Electrodynamics. Check "Liénard-Wiechert potentials" if you are curious).