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Quantenmechanik, Herbstsemester 2023

Blatt 11 (Zusatzpunkte)

Abgabe: 5.12.23, 12:00H (Treppenhaus 4. Stock)
Tutor: Manel Bosch, Zi.: 2.12

Schriftlicher Test: Dienstag, 19. Dezember 2023, 10.15 - 12 Uhr
Hilfsmittel: Ein beidseitiges **hand**beschriebenes A4 Blatt.

(1) **Born approximation** (3 Bonuspunkte)

A particle of mass m is scattered at the potential

$$V(\mathbf{r}) = \begin{cases} -V_0 & r < a, \\ 0 & r \geq a \end{cases} ,$$

here, $r = |\mathbf{r}|$, and V_0 can be positive or negative.

(a) Calculate the differential cross section in the (first) Born approximation.

(b) Discuss the limit of low energy $ka \ll 1$ where $k = \sqrt{2mE/(\hbar^2)}$. Calculate the total cross section in this limit.

(c) Discuss the validity of the Born approximation for this example.

(2) **Sakurai Fig. 7.8** (2 Bonuspunkte)

The figure shown below is taken from Sakurai [see p. 412 (436) in the old (new) edition]. It is supposed to illustrate the concept of the scattering phase for scattering at a square-well potential. The figure is imprecise/incorrect in several places.
Redraw the figure **carefully** yourselves, correcting the mistakes/imprecisions.

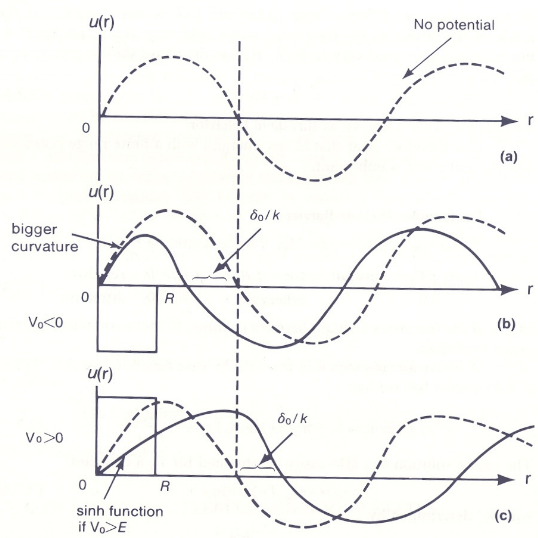

FIGURE 7.8. Plot of $u(r)$ versus r. (a) For $V = 0$ (dashed line). (b) For $V_0 < 0$, $\delta_0 > 0$ with the wave function (solid line) pulled in. (c) For $V_0 > 0$, $\delta_0 < 0$ with the wave function (solid line) pulled out.

(3) **Spherical δ-function shell** (5 Bonuspunkte)

Consider the case of low-energy scattering from a spherical δ-function shell,

$$V(r) = \alpha\delta(r - a) \; ,$$

where α, a are positive constants.

(a) Find the s-wave ($\ell = 0$) scattering phase shift $\delta_0(k)$. Express your answer in terms of the dimensionless quantity $\beta := 2ma\alpha/\hbar^2$.

(b) Calculate the scattering amplitude $f(\theta)$, the differential cross-section $d\sigma/d\Omega(\theta)$, and the total cross section σ. Assume $ka \ll 1$ such that only the $\ell = 0$ term contributes significantly (i.e., neglect the terms with $\ell \neq 0$).

(4) **Hard-sphere scattering** (5 Bonuspunkte)

Consider scattering of a particle of energy $E = \hbar^2 k^2/2m$ by the potential

$$V(r) = \begin{cases} \infty & r < a, \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the scattering phase shifts δ_l, the partial scattering amplitudes f_l, and the partial (total) cross sections σ_l.
Hint: Use the ansatz $R_l(r) = \frac{1}{2}[e^{2i\delta_l} h_l(kr) + h_l^*(kr)]$ for the radial part of the wavefunction.

(b) Determine δ_0 and $R_0(r)$. Plot $R_0(r)$. Hint: $j_0(x) = \sin(x)/x$, $n_0(x) = -\cos(x)/x$.

(c) Consider the limit of low energies, $ka \ll 1$, and calculate the partial scattering amplitude f_0. For $l \neq 0$, show that $\lim_{ka\to 0} \sin\delta_l/k = 0$ and conclude that $f_l \to 0$. Discuss the differential and total cross section in the low-energy limit.
Hint: For $x \to 0$, $j_l(x) = x^l/1 \cdot 3 \cdot \ldots (2l+1)$, $n_l(x) = -1 \cdot 3 \cdot \ldots (2l-1)/x^{l+1}$

(d) * Show that in the limit of high energies, $ka \gg 1$, the total cross section is $\sigma_{\text{tot}} = 2\pi a^2$, i.e., twice the geometric cross section. Explanation?