Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 207 36 92, Christoph.Bruder@unibas.ch)

Quantenmechanik, Herbstsemester 2023

Blatt 1

Abgabe: 27.09.23, 12:00H (auf adam oder Treppenhaus 4. Stock)
Tutor: Niels Lörch, Zimmer 4.10

Die **Übungskreditpunkte** erhält, wer sowohl 50% der Punkte aus den Hausaufgaben erreicht als auch 50% der Punkte aus dem schriftlichen Test am Ende des Semesters.

(1) **Spin 1 system** (3 Punkte)

Consider a spin 1 system. The spin matrices are

$$S_x = \hbar\sqrt{\frac{1}{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \; S_y = \hbar\sqrt{\frac{1}{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}, \text{ and } S_z = \hbar\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

(a) What are the possible measurement results if S_z is measured?

(b) Assume the system is prepared in an eigenstate of S_z such that a measurement of S_z yields $\hbar$. In this state, what are the expectation values $\langle S_y\rangle$ and $\langle S_y^2\rangle$? Discuss your results.

(c) Now assume the system to be prepared in the eigenstate of S_z with the eigenvalue 0. What are the possible outcomes and their probabilities when S_y is measured? Hint: the normalized eigenvectors of S_y are

$$\frac{1}{2}\begin{pmatrix}-1\\ \mp i\sqrt{2}\\ 1\end{pmatrix} \text{ with eigenvalues } \pm\hbar, \quad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\\ 1\end{pmatrix} \text{ with eigenvalue } 0\,.$$

(2) **Properties of the density operator** (2 Punkte)

The density operator is defined as $\hat\rho = \sum_{i=1}^{N} p_i |\psi_i\rangle\langle\psi_i|$ where $0 \leq p_i \leq 1$ are the probabilities that the system is in state $|\psi_i\rangle$. The $|\psi_i\rangle$ do not need to be orthogonal.

(a) Show that $\mathrm{Tr}\,\hat\rho = 1$.

(b) Show that $\mathrm{Tr}\,\hat\rho^2 = 1$ if and only if ρ is pure.
Hint: Write down the expression for $\mathrm{Tr}\,\hat\rho^2$ and distinguish the two cases that only one or at least two different $|\psi_i\rangle$'s contribute to $\hat\rho$.

(3) **Spin 1/2 continued** (5 Punkte)

We continue to consider a particle with spin $\frac{1}{2}$. The notation is the same as in problem 1 from Blatt 0.

(a) Write the projector onto $|\downarrow\rangle_x$ as a matrix in the z-basis.

(b) Let the spin-$\frac{1}{2}$ particle be with probability $\frac{2}{3}$ in the state $|\uparrow\rangle_y$ and with probability $\frac{1}{3}$ in the state $|\downarrow\rangle_x$ (Note the indices x and y!). Give a representation of the density operator $\hat{\rho}$. Is this a mixed or a pure state?

(c) What is the probability to measure $\pm\frac{\hbar}{2}$ on measuring $\hat{S}_x$ in a system described by $\hat{\rho}$ from problem (b).

(d) Can you find a state $|\psi\rangle$ that reproduces the measurement results in (c)?

(e) What is the state after the measurement in (c) and (d)?

(f) Compute the expectation value of $\hat{S}_y$ using $\hat{\rho}$ from problem (b) and $|\psi\rangle$ from problem (d).