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# Quantenmechanik, Herbstsemester 2023

**Blatt 0**

Präsenzübung am 21. September 2023

(1) **Spin 1/2**
Consider a particle with spin $\frac{1}{2}$. Before each of the measurements discussed below, the particle has been prepared in the state

$$|\psi\rangle = i\frac{1}{\sqrt{5}}\,|\uparrow\rangle_z - \frac{2}{\sqrt{5}}\,|\downarrow\rangle_z\,.$$

Here, $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$ are the eigenstates of the operator $\hat{S}_z$ with eigenvalues $\frac{\hbar}{2}$, $-\frac{\hbar}{2}$, respectively.

NB: In the $z$-basis, we have $\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$, and $\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$.

(a) What is the probability to obtain the value $-\frac{\hbar}{2}$ when measuring $\hat{S}_z$?

(b) Compute the expectation value of $\hat{S}_z$.

(c) Compute the expectation value of $\hat{S}_x$ and $\hat{S}_y$.

(d) Rewrite the vector $|\uparrow\rangle_x$, which is the eigenstate of $\hat{S}_x$ with eigenvalue $\frac{\hbar}{2}$, as a linear combination of $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$. Same for $|\downarrow\rangle_x$

(e) What is the probability to obtain the value $\pm\frac{\hbar}{2}$ when $\hat{S}_x$ is measured? Check that your result is consistent with (c).

(f) Repeat (d) and (e) for $\hat{S}_y$.

(g) What is the probability to obtain the value 0 when measuring $\hat{S}_x$?