

Mechanik, Herbstsemester 2022

Blatt 9

Abgabe: 22.11.2022, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**

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(1) **Symmetric top in a uniform gravitational field (“heavy top”)** (4 Punkte)

In the lecture we discussed the motion of a symmetric top placed in a uniform gravitational field and fixed in one point by expressing the Lagrangian in terms of Euler angles,

$$L(\phi, \theta, \psi, \dot{\phi}, \dot{\theta}, \dot{\psi}) = \frac{1}{2}I_1(\dot{\phi}^2 \sin^2 \theta + \dot{\theta}^2) + \frac{1}{2}I_3(\dot{\phi} \cos \theta + \dot{\psi})^2 - mgl \cos \theta.$$

- Assume that the top rotates around the vertical axis of the lab system, $\theta = 0$. Show that in this case, $L_z = L_3$ where $L_z = p_\phi = \partial L / \partial \dot{\phi}$ is the angular momentum about the vertical axis and $L_3 = p_\psi = \partial L / \partial \dot{\psi}$ the angular momentum about the body axis x_3 .
- Plot the effective potential for a representative choice of parameters. Find the condition under which the vertical rotation is *stable* under small perturbations θ . Hint: expand the effective potential around $\theta = 0$.
- Bonus points: solve the equations of motions numerically and confirm the nutation behavior discussed in the lecture and the stability condition discussed in (b).

(2) **Hamiltonians for well-known problems** (6 Punkte)

Starting from the Lagrangian L , calculate the Hamiltonian H and write down Hamilton’s equations of motion.

- One-dimensional harmonic oscillator
- Three-dimensional harmonic oscillator
- Charged particle in a magnetic field
- Particle in a central potential.

Hint: $H = H(r, \phi, p_r, p_\phi)$ where $p_r = \frac{\partial L}{\partial \dot{r}}$ and $p_\phi = \frac{\partial L}{\partial \dot{\phi}}$ are the canonical momenta associated with the (standard) polar coordinates r and ϕ .

- Double pendulum (see Blatt 7 problem 1(a) for L).
Result for H :

$$H(\phi_1, \phi_2, p_{\phi_1}, p_{\phi_2}) = -ga(m_1 + m_2) \cos(\phi_1) - gam_2 \cos(\phi_2) + \frac{m_2 p_{\phi_1}^2 + (m_1 + m_2) p_{\phi_2}^2 - 2m_2 p_{\phi_1} p_{\phi_2} \cos(\phi_1 - \phi_2)}{a^2 m_2 (2m_1 + m_2 - m_2 \cos(2(\phi_1 - \phi_2)))}.$$

Hint: Write L as

$$L(\phi, \dot{\phi}) = \frac{1}{2} \dot{\phi}^T A \dot{\phi} - V(\phi),$$

where A is a symmetric positive-definite matrix.