Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 207 36 92, Christoph.Bruder@unibas.ch)

Mechanik, Herbstsemester 2022

Blatt 3

Abgabe: 11.10.2022, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
Tutor: Tobias Kehrer Zi.: 4.48; tobias.kehrer@unibas.ch

(1) **Principle of least action for a particle under a constant force** (2 Punkte)
A particle is subjected to the potential $V(x) = -Fx$ where F is constant. The particle travels from $x = x_0 = 0$ to $x = x_1$ in a time t_1. Use the ansatz $x(t) = A + Bt + Ct^2$ and find the values of A, B, and C such that the action is a minimum.

(2) **Charge in crossed electric and magnetic fields** (2 Punkte)
Consider time-independent and homogeneous electric and magnetic fields; $\mathbf{E}$ points in x-direction and $\mathbf{B}$ in z-direction. An electron is injected with velocity v in y-direction.

(a) Find a vector potential $\mathbf{A}$ such that $\mathbf{B} = (0, 0, B) = \nabla \times \mathbf{A}$. Write down Lagrange's equation for the system.

(b) Solve Lagrange's equation and plot and discuss the motion of the electron in detail.

(3) **Minimal surface** (3 Punkte)
Consider a surface defined by rotating the curve $r = f(z)$ about the z-axis; we further assume $z \in [-1, 1]$ and $f(1) = f(-1) = R$.

(a) Express the area of the surface as an integral.

(b) Finde the function f that minimizes the area.
Hint: Use the "first integral" of the Euler-Lagrange equation discussed in the lecture.
Remark: a soap film clamped to two circles of radius R will assume this shape.

(c) Discuss your solution carefully. Is it compatible with all values of R? If not, can you find a minimal surface for this case?

(4) **Bead on a stick** (3 Punkte)
(continuation of the example discussed in the lecture)
A stick is pivoted a the origin and is arranged to swing around in the xy-plane at constant angular velocity ω. A bead of mass m slides frictionless along the stick. Let r be the radial position of the bead. Show that the quantity

$$E = \sum_{i=1}^{f} \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

is conserved. Explain why this is *not* the energy of the bead.

(5) **Minimum or saddle** (3 Bonuspunkte)

Consider a one-dimensional harmonic oscillator that has the Lagrangian

$$L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} m \omega^2 x^2 \, .$$

Let $x_0(t)$ be a function that yields a stationary value of the action. Then we know that $x_0(t)$ fulfills the Euler-Lagrange equation, $m\ddot{x}_0 = -m\omega^2 x_0$. Consider a slight variation on this path, $x_0(t) + \eta(t)$, where $\eta(t_1) = \eta(t_2) = 0$.

(a) Calculate the action $S[x_0 + \eta]$ for $t_1 = 0$, $t_2 = \tau$.

(b) Show that it is always possible to find a function η such that $S[x_0 + \eta] - S[x_0]$ is positive. Conclude that $S[x_0]$ can never be a maximum of the action.

(c) Find a combination of τ and $\eta(t)$ such that $S[x_0 + \eta] - S[x_0]$ is negative. Conclusion?