Departement Physik, Universität Basel
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Mechanik, Herbstsemester 2022

Blatt 11 = letztes "offizielles" Übungsblatt
(Blatt 12 = Bonus-Übungsblatt)

Abgabe: 6.12.2022, 12:00H, **entweder auf adam in den entsprechenden Ordner, oder in das Fach im Treppenhaus 4. Stock!**
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(1) **Properties of the Poisson bracket** (2 Punkte)
The Poisson bracket of two observables (i.e., functions on phase space) $f(\mathbf{q}, \mathbf{p}; t)$ and $g(\mathbf{q}, \mathbf{p}; t)$ is defined as

$$\{f, g\} \equiv \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$$

where n is the number of degrees of freedom.

(a) Write down the equations of motion of the one-dimensional harmonic oscillator using the Poisson bracket.

(b) Show that the components of the angular momentum vector $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ fulfill $\{L_x, L_y\} = L_z$ (and cyclic permutations).

(2) **Police chases criminals** (2 Punkte)
As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$.

Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?

(3) **Simultaneity** (3 Punkte)
Event A happens at $\mathbf{x}_A = (5, 3, 0)$ and at time $ct_A = 15$; event B occurs at $\mathbf{x}_B = (10, 8, 0)$ and $ct_B = 5$, both in system K.

(a) Calculate the invariant interval $I \equiv c^2(t_A - t_B)^2 - (\mathbf{x}_A - \mathbf{x}_B)^2$ between A and B. Why is it the same in all inertial systems?

(b) Is there an inertial system K' in which they occur simultaneously? If so, find its velocity (magnitude and direction) relative to K.

(c) Is there an inertial system K'' in which they occur at the same point? If so, find its velocity relative to K.

Repeat (a) – (c) for $\mathbf{x}_A = (2, 0, 0)$, $ct_A = 1$; and $\mathbf{x}_B = (5, 0, 0)$, $ct_B = 3$.

(4) **Communicating with rockets** (3 Punkte)

A rocket ship leaves earth at a speed of $\frac{3}{5}c$. When a clock on the rocket (system K') says 1 hour has elapsed, the rocket ship sends a light signal back to earth (system K).

(a) According to *earth* clocks, when was the signal sent?

(b) According to *earth* clocks, how long after the rocket left did the signal arrive back on earth?

(c) According to the *rocket* observer, how long after the rocket left did the signal arrive back on earth?