

Quantenmechanik, Herbstsemester 2021

Blatt 1

Abgabe: 28.09.21, 12:00H (Treppenhaus 4. Stock)

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Die Übungskreditpunkte erhält, wer sowohl 50% der Punkte aus den Hausaufgaben erreicht als auch 50% der Punkte aus dem schriftlichen Test am Ende des Semesters.

(1) **Spin 1 system** (3 Punkte)

Consider a spin 1 system. The spin matrices are

$$S_x = \hbar \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \hbar \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \text{and} \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- (a) What are the possible measurement results if S_z is measured?
- (b) Assume the system is prepared in an eigenstate of S_z such that a measurement of S_z yields \hbar . In this state, what are the expectation values $\langle S_y \rangle$ and $\langle S_y^2 \rangle$? Discuss your results.
- (c) Now assume the system to be prepared in the eigenstate of S_z with the eigenvalue 0. What are the possible outcomes and their probabilities when S_y is measured? Hint: the normalized eigenvectors of S_y are

$$\frac{1}{2} \begin{pmatrix} -1 \\ \mp i\sqrt{2} \\ 1 \end{pmatrix} \text{ with eigenvalues } \pm \hbar, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \text{ with eigenvalue } 0.$$

(2) **Properties of the density operator** (2 Punkte)

The density operator is defined as $\hat{\rho} = \sum_{i=1}^N p_i |\psi_i\rangle \langle \psi_i|$ where $0 \leq p_i \leq 1$ are the probabilities that the system is in state $|\psi_i\rangle$. The $|\psi_i\rangle$ do not need to be orthogonal.

- (a) Show that $\text{Tr } \hat{\rho} = 1$.
- (b) Show that $\text{Tr } \hat{\rho}^2 = 1$ if and only if ρ is pure.
Hint: Write down the expression for $\text{Tr } \hat{\rho}^2$ and distinguish the two cases that only one or at least two different $|\psi_i\rangle$'s contribute to $\hat{\rho}$.

(3) **Spin 1/2 continued**

(5 Punkte)

We continue to consider a particle with spin $\frac{1}{2}$. The notation is the same as in problem 1 from Blatt 0.

- (a) Write the projector onto $|\downarrow\rangle_x$ as a matrix in the z -basis.
- (b) Let the spin- $\frac{1}{2}$ particle be with probability $\frac{1}{3}$ in the state $|\uparrow\rangle_y$ and with probability $\frac{2}{3}$ in the state $|\downarrow\rangle_x$ (Note the indices x and y !). Give a representation of the density operator $\hat{\rho}$. Is this a mixed or a pure state?
- (c) What is the probability to measure $\pm\frac{\hbar}{2}$ on measuring \hat{S}_x in a system described by $\hat{\rho}$ from problem (b).
- (d) Can you find a state $|\psi\rangle$ that reproduces the measurement results in (c)?
- (e) What is the state after the measurement in (c) and (d)?
- (f) Compute the expectation value of \hat{S}_y using $\hat{\rho}$ from problem (b) and $|\psi\rangle$ from problem (d).