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Mechanik, Herbstsemester 2020

Blatt 8

Abgabe: 10.11.2020, 12:00H, **auf adam in den entsprechenden Ordner. Ein File pro Abgabe; der Filename muss Ihren Namen enthalten, sonst wird nicht korrigiert!**
Tutor: Gaomin Tang, gaomin.tang@unibas.ch

(1) **Bottle on the floor of a tram** (2 Punkte)
A bottle (modeled as a homogeneous circular cylinder of radius R and mass M) lies on the floor of a tram; its orientation is perpendicular to the tram's direction of motion. When the tram starts moving with acceleration a, the bottle will start to roll (we assume without sliding).

(a) Calculate the moment of inertia I along the symmetry axis. Result: $I = \frac{1}{2}MR^2$

(b) Determine the acceleration $\tilde{a}$ of the bottle relative to the passengers.

(2) **Inertia ellipsoid** (3 Punkte)
Consider a body with density $\rho(\mathbf{x})$ and inertia tensor $I_{\mu\nu}$. We define the set Γ of all points $\boldsymbol{\omega}$ for which $\boldsymbol{\omega}^T I \boldsymbol{\omega} = 1$.

(a) Show that Γ is an ellipsoid whose axes are parallel to the principle axes of the body and whose (semi-)axes lengths are $1/\sqrt{I_i}$ where I_i, $i = 1, 2, 3$ are the principle moments of inertia of the body.

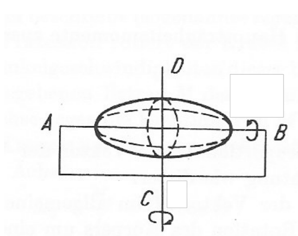
(b) Assume that the body is invariant under a symmetry transformation R, $\rho(R\mathbf{x}) = \rho(\mathbf{x})$. Show that Γ is also invariant under R. Hint: $R^{\mathrm{T}} R = 1$.

(c) Show: if the body has a k-fold symmetry axis (i.e., is invariant under rotations by $2\pi/k$ about this axis) with $k \geq 3$, this axis is a principle axis and the body is a symmetric top. If the body has more than one such axis, it is a spherical top.

(3) **Rotating ellipsoid** (2 Punkte)
Consider a homogeneous three-axial ellipsoid with principal moments of inertia I_1, I_2, and I_3 that rotates about one of its axes (AB in the figure) with constant angular velocity $\dot{\alpha}$. This axis rotates itself about the direction CD that is fixed in space, perpendicular to AB, and passes through the center of mass of the ellipsoid, with constant angular velocity $\dot{\beta}$.

(a) Find $\boldsymbol{\omega}$ in the body's coordinate system by projecting the rotations onto the principal axes.

(b) Write down the kinetic energy. Show that it is not constant – interpretation?

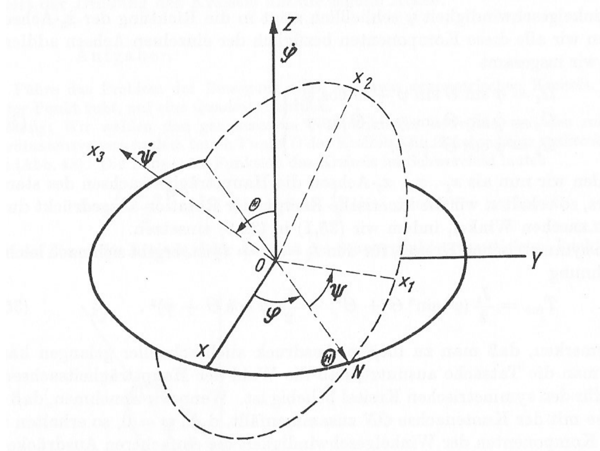


(4) **Angular velocity in the body system and Euler angles** (3 Punkte)

The angular velocity $\boldsymbol{\omega}$ in the body frame can be expressed in terms of the time-dependent Euler angles by projecting the Euler rotations onto the body axes. Show that

$$\boldsymbol{\omega} = \begin{pmatrix} \dot\varphi \sin\theta \sin\psi + \dot\theta \cos\psi \\ \dot\varphi \sin\theta \cos\psi - \dot\theta \sin\psi \\ \dot\varphi \cos\theta + \dot\psi \end{pmatrix} .$$

Hint: $\boldsymbol{\omega} = \boldsymbol{\omega}_\varphi + \boldsymbol{\omega}_\theta + \boldsymbol{\omega}_\psi$, where $\boldsymbol{\omega}_\varphi$, $\boldsymbol{\omega}_\theta$, and $\boldsymbol{\omega}_\psi$ are the angular velocity vectors that correspond to the Euler rotations. Project them onto the body axes. E.g., $\boldsymbol{\omega}_\psi$ points in direction x_3, hence $\boldsymbol{\omega}_\psi = (0, 0, \dot\psi)$.



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(5) **Bonus problem: Static and dynamic unbalance** (4 Punkte)

A cylinder is fixed on an axis that rotates with the constant angular velocity ω, see figure. The center of mass S of the cylinder has distance e from the axis; the cylinder axis and the rotation axis enclose an angle θ.

This type of rotation leads to two kinds of unbalances since (i) the center of mass does not lie on the rotation axis (*static unbalance*). (ii) the cylinder axis is not parallel to the rotation axis (*dynamic unbalance*).

Calculate the forces in the bearings A and B.

Hint: Use Euler's equations and the fact that the center of mass is accelerated as if the sum of the external forces would be applied to it.