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Mechanik, Herbstsemester 2020

Blatt 3

Abgabe: 6.10.2020, **bitte auf adam in den entsprechenden Ordner!**
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(1) **Principle of least action for a particle under a constant force** (2 Punkte)
A particle is subjected to the potential $V(x) = -Fx$ where F is constant. The particle travels from $x = x_0 = 0$ to $x = x_1$ in a time t_1. Use the ansatz $x(t) = A + Bt + Ct^2$ and find the values of A, B, and C such that the action is a minimum.

(2) **Charge in crossed electric and magnetic fields** (2 Punkte)
Consider time-independent and homogeneous electric and magnetic fields; $\mathbf{E}$ points in x-direction and $\mathbf{B}$ in z-direction. An electron is injected with velocity v in y-direction.

- (a) Find a vector potential $\mathbf{A}$ such that $(0, 0, B) = \nabla \times \mathbf{A}$. Write down Lagrange's equation for the system.
- (b) Solve Lagrange's equation and plot and discuss the motion of the electron in detail.

(3) **Oscillations on a cycloid curve** (3 Punkte)
A particle subject to the homogenous gravitational field ($\mathbf{g} = -g\,\mathbf{e}_y$) is constrained to move (without friction) on a cycloid defined by the parametric representation
$x = R(\varphi - \sin\varphi)$, $y = -R(1 - \cos\varphi)$ where $\varphi \in [0, 2\pi]$.

- (a) Sketch the cycloid.
- (b) Choose $\xi = 4R\cos(\frac{\varphi}{2})$ as a generalized coordinate. Write down Lagrange's equation and the equation of motion of the problem.
- (c) Solve the equation of motion and calculate the period of the oscillation. Show that the period is independent of the amplitude of the oscillation.
  Bonus question: Is this common – can you find other curves for which this is true?

(4) **Minimal surface** (3 Punkte)
Consider a surface defined by rotating the curve $r = f(z)$ about the z-axis; we further assume $z \in [-1, 1]$ and $f(1) = f(-1) = R$.

- (a) Express the area of the surface as an integral.
- (b) Finde the function f that minimizes the area.
  Hint: Use the "first integral" of the Euler-Lagrange equation discussed in the lecture.
  Remark: a soap film clamped to two circles of radius R will assume this shape.
- (c) Discuss your solution carefully. Is it compatible with all values of R? If not, can you find a minimal surface for this case?