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Mechanik, Herbstsemester 2020

Bonus-Blatt 13

Abgabe: 15.12.2020 **auf adam in den entsprechenden Ordner. Ein File pro Abgabe; der Filename muss Ihren Namen enthalten, sonst wird nicht korrigiert!**

(1) **Lorentz-transformed angle** (2 Bonus-Punkte)
A sailboat is manufactured so that the mast leans at an angle θ' with respect to the deck. An observer standing on a dock sees the boat go by at speed v. What angle does this observer say the mast makes?

(2) **General velocity addition rule** (5 Bonus-Punkte)
In the lecture we discussed the velocity addition rule by combining two Lorentz transformations. Here, we want to rederive it by using the 4-velocity $v^{\,\alpha} = \gamma(c, \mathbf{u})$ which is a 4-vector; $\mathbf{u}$ is the 3-velocity and $\gamma = (1 - u^2/c^2)^{-1/2}$.
A particle moves with 3-velocity $\mathbf{u}_2$ in K' and K' moves with velocity u_1 along the x-axis of K. Determine the components of the 3-velocity $\mathbf{u}$ of the particle in K and show that $\mathbf{u}$ can be written as

$$\mathbf{u} = \frac{\mathbf{u}_1 + \mathbf{u}_{2\parallel} + \mathbf{u}_{2\perp}\sqrt{1 - u_1^2/c^2}}{1 + \mathbf{u}_1\mathbf{u}_2/c^2}$$

Here, $\mathbf{u}_2 = \mathbf{u_{2\parallel}} + \mathbf{u_{2\perp}}$ where $\mathbf{u_{2\parallel}}$ is parallel to $\mathbf{u}_1$ and $\mathbf{u_{2\perp}}$ perpendicular to $\mathbf{u}_1$. Discuss the cases $\mathbf{u}_1 \parallel \mathbf{u}_2$ and $\mathbf{u}_1 \perp \mathbf{u}_2$.
Hint: Find the 4-velocity $v'^{\,\alpha}$ of the particle in K'. Lorentz-backtransform to K to find the 4-velocity v^α in K. Use the definition of the 4-velocity to calculate $\mathbf{u}$.

(3) **Relativistic kinematics** (3 Bonus-Punkte)

(a) A particle of mass m whose total energy is twice its rest energy collides with an identical particle at rest. If they stick together, what is the mass of the resulting composite particle? What is its velocity?

(b) A neutral pion of (rest) mass m and (relativistic) momentum $p = \frac{3}{4}mc$ decays into two photons. One of the photons is emitted in the same direction as the original pion, and the other in the opposite direction. Find the (relativistic) energy of each photon.