Departement Physik, Universität Basel
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Mechanik, Herbstsemester 2020

Blatt 1

Abgabe: 22.9.2020, 12:00H per email
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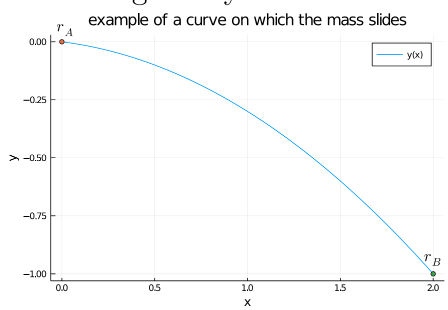
Die **Übungskreditpunkte** erhält, wer 50% der Punkte aus den Hausaufgaben erreicht.

(1) **How quickly can a mass slide from r_A to r_B?** (6 Punkte)
We consider a point mass that slides without friction on a curve $y(x)$ in the xy-plane connecting the two points $r_A = (0, 0)$ and $r_B = (2, -1)$. The point mass starts at r_A with velocity 0 and is subject to the Earth's gravitational field that is assumed to be homogeneous and point in the negative y-direction.



(a) Use energy conservation to calculate the velocity of the particle at a given y-coordinate. Result: $v = \sqrt{2g(-y)}$.
Show that the total time that the particle needs to reach r_B can be expressed as $T = \int_{r_{Ax}}^{r_{Bx}} \mathrm{d}x \frac{\sqrt{1+y'^2}}{\sqrt{2g(-y)}}$.

(b) Calculate the time T exactly if $y(x)$ is a straight line. Result: $T_{\text{straight}} = \sqrt{10/9.81}$s.

(c) Write a computer program (using Julia or some other programming language) to calculate T for an arbitrary curve $y(x)$. Confirm that you obtain the result of (b) in the case of a straight line. Now try modifications of a straight line and explore curves for which $T < T_{\text{straight}}$. What is the minimal time that you can find??

(2) **Velocity and acceleration in polar and spherical coordinates** (4 Punkte)
In a Cartesian coordinate system, the basis vectors $\mathbf{e}_x$, $\mathbf{e}_y$, $\mathbf{e}_z$ are spatially independent. In curvilinear coordinate systems, the basis vectors are generally spatially dependent, e.g., in polar coordinates (ρ, ϕ), they take the form: $\mathbf{e}_\rho = (\cos\phi, \sin\phi)$, $\mathbf{e}_\phi = (-\sin\phi, \cos\phi)$. For a moving particle, the basis vectors will therefore effectively depend on time t.

(a) Calculate velocity and acceleration for the trajectory $\mathbf{r}(t) = \rho(t)\mathbf{e}_\rho(t)$ in polar coordinates.

(b) Write down the basis vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ for the spherical coordinate system (r, θ, ϕ). Repeat (a) for a trajectory $\mathbf{r}(t) = r(t)\mathbf{e}_r(t)$ in the spherical coordinate system.