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Quantenmechanik, Herbstsemester 2019

Blatt 9

Abgabe: 19.11.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Gaomin Tang, Zi.: 4.16

(1) **Matrix elements of z** (2 Punkte)

In the discussion of the Stark effect in the lecture we used some properties of the matrix elements of z with the H-atom states $|nlm\rangle$ that we want to prove now.

(a) Show that $[L_z, z] = 0$ and conclude that $\langle n'l'm'|z|nlm\rangle = 0$ unless $m' = m$.

(b) Use a symmetry argument to prove that $\langle n'lm'|z|nlm\rangle = 0$ (same l!).

(c) Show that $\langle 200|z|210\rangle = -3a_0$ where $a_0 = 4\pi\epsilon_0\hbar^2/(me^2)$ is the Bohr radius.
Hint: the H-atom wave functions are $\psi_{nlm}(r,\theta,\phi) = \langle r,\theta,\phi|nlm\rangle = R_{nl}(r)Y_{lm}(\theta,\phi)$.
In particular, $R_{20}(r) = 2\left(\frac{1}{2a_0}\right)^{3/2}(1-\frac{r}{2a_0})e^{-\frac{r}{2a_0}}$ and $R_{21}(r) = \frac{1}{\sqrt{3}}\left(\frac{1}{2a_0}\right)^{3/2}\frac{r}{a_0}e^{-\frac{r}{2a_0}}$

(2) **Perturbed two-dimensional harmonic oscillator** (5 Punkte)

Consider the two-dimensional harmonic oscillator with a perturbed potential energy of the form

$$V(x,y) = \frac{1}{2}m\omega^2(x^2 + y^2 + \lambda xy)\,. \qquad (1)$$

(a) Calculate the energy eigenvalues for the unperturbed case ($\lambda = 0$) and discuss their degeneracies.
Hint: Use creation (annihilation) operators for each of the two degrees of freedom x, y, i.e.,

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_1 + a_1^\dagger), \qquad p_x = i\sqrt{\frac{m\hbar\omega}{2}}(a_1^\dagger - a_1)\,,$$

and similarly for y, p_y, and a_2, $a_2^\dagger$.

(b) Compute the ground-state energy of the system ($\lambda \neq 0$) up to second order in λ and the ground-state wave function up to first order.

(c) The first excited state of the unperturbed system ($\lambda = 0$) is doubly degenerate. Calculate the energy splitting up to first order in λ. What are the corresponding eigenstates in zeroth order?

(d) * Compare with the exact result.
Hint: express (1) as a sum of two harmonic potentials.

(3) **Time-dependent perturbation of a harmonic oscillator** (3 Punkte)

A homogeneous electric field $E(t) = E_0 \exp(-t^2/\tau^2) \cos \omega_0 t$ is applied to a charged one-dimensional oscillator $H = \dfrac{p^2}{2m} + \dfrac{1}{2} m\omega^2 x^2$.

(a) Assume that the oscillator was prepared in the n^{th} excited state before the field was turned on (i.e., for $t \to -\infty$). Use first-order perturbation theory to calculate the transition probability to find the oscillator in the k^{th} excited state for $t \to +\infty$.

(b) Discuss your result for the relevant limits of ω_0 and τ.