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Quantenmechanik, Herbstsemester 2019

Blatt 4

Abgabe: 15.10.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Michal Kloc, Zi. 4.10

(1) **Unequal time commutation relations** (3 Punkte)

Calculate the commutation relations $[\hat{x}_H(t), \hat{p}_H(t')]$ of the (one-dimensional) position and momentum operators in the Heisenberg picture at times t, t' for the following cases

(a) a particle acted on by a constant force

(b) a harmonic oscillator.

(2) **Time evolution of a free particle** (2 Punkte)

Consider a free particle in three dimensions, $\hat{H} = \hat{\mathbf{p}}^2/2m$. Calculate the commutator $[\hat{x}_{jH}(t), \hat{x}_{jH}(0)]$ of the position operator $\hat{\mathbf{x}}_H$ in the Heisenberg picture, here, $\hat{x}_{jH}$, $j = 1, 2, 3$ are the components of $\hat{\mathbf{x}}_H$.
Give a lower bound for $\Delta\hat{x}_{jH}(t)\,\Delta\hat{x}_{jH}(0)$ and interpret your result.
$\Delta A := \sqrt{\langle A^2\rangle - \langle A\rangle^2}$

(3) **Harmonic oscillator I** (2 Punkte)

The harmonic oscillator $H = (1/2m)p^2 + (1/2)m\omega^2x^2$ can be written in the form $H = \hbar\omega(a^\dagger a + 1/2)$, where $a^\dagger$ and a are the creation and annihilation operators (also called ladder operators) defined in the lecture. $H|n\rangle = \hbar\omega(n + 1/2)|n\rangle$.
Use the ladder operators to calculate $\langle n'|p|n\rangle$, $\langle n'|xp|n\rangle$, $\langle n'|px|n\rangle$, $\langle n'|x^2|n\rangle$, and $\langle n'|p^2|n\rangle$.
What is the uncertainty product $\Delta x\Delta p$ in eigenstate $|n\rangle$?

(4) **Harmonic oscillator II** (3 Punkte)

Consider a harmonic oscillator of mass m and angular frequency ω. At time $t = 0$, the state of this oscillator is given by $|\psi(0)\rangle = \sum_n c_n|n\rangle$ where the $|n\rangle$ are eigenstates with energies $\hbar\omega(n + 1/2)$ for $n \geq 0$.

(a) What is the probability W that a measurement of the oscillator's energy performed at an arbitrary time $t > 0$, will yield a result greater than $3\hbar\omega$? When $W = 0$, what are the non-zero coefficients c_n?

(b) From now on, assume that only c_0 and c_2 are different from zero. Write the normalization condition for $|\psi(0)\rangle$ and the expectation value $\bar{E}$ of the energy in terms of c_0 and c_2. Calculate $|c_0|^2$ and $|c_2|^2$ if $\bar{E} = \hbar\omega$.

(c) If at time $t = 0$ the state of the oscillator is $|\psi(0)\rangle = \dfrac{1}{\sqrt{5}}(2|1\rangle + |2\rangle)$, calculate $|\psi(t)\rangle$ for $t > 0$ and the mean value $\langle x(t)\rangle$ of the position at t.