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Quantenmechanik, Herbstsemester 2019

Blatt 11

Abgabe: 3.12.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Michal Kloc, Zi.: 4.10

(1) **Wigner-Eckart theorem** (3 Punkte)
Electromagnetic quadrupole transitions in the hydrogen atom are described by matrix elements of the (spherical) quadrupole operators $Q_m^{(2)} \sim r^2 Y_{2m}$ that form a set of spherical tensor operators.

(a) Calculate the ratio B/A of the following matrix elements; here, $|nlm\rangle$ are the eigenstates of the hydrogen atom:

$$A = \langle n'43|Q_2^{(2)}|n21\rangle \,,$$
$$B = \langle n'4,-2|Q_0^{(2)}|n2,-2\rangle \,.$$

(b) Calculate

$$C = \langle n'51|Q_2^{(2)}|n1,-1\rangle \,,$$
$$D = \langle n'31|Q_0^{(2)}|n1,-1\rangle \,.$$

(c) Consider the matrix element $\langle 4lm|z(x+iy)|n21\rangle$ where x,y,z are Cartesian coordinates. Which values for l and m are allowed, i.e., lead to non-vanishing values?

(2) **Classical scattering** (2 Punkte)
Consider a classical point particle incident on some scattering center described by a central potential.

(a) Assume that the impact parameter (perpendicular distance between the center of force and the incident velocity) is b. Show that the differential cross section can be expressed as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\theta) = \frac{b}{\sin\theta}\left|\frac{\mathrm{d}b}{\mathrm{d}\theta}\right| \,.$$

(b) Use (a) to calculate $\mathrm{d}\sigma/\mathrm{d}\Omega$ for a hard-sphere scattering potential (radius R). Calculate the total cross section.

(3) **Born approximation** (3 Punkte)
A particle of mass m is scattered at the potential

$$V(\mathbf{r}) = \begin{cases} -V_0 & r < a, \\ 0 & r \geq a \end{cases} \,,$$

here, $r = |\mathbf{r}|$, and V_0 can be positive or negative.

(a) Calculate the differential cross section in the (first) Born approximation.

(b) Discuss the limit of low energy $ka \ll 1$ where $k = \sqrt{2mE/(\hbar^2)}$. Calculate the total cross section in this limit.

(c) Discuss the validity of the Born approximation for this example.

(4) **Sakurai Fig. 7.8** (2 Punkte)
The figure shown below is taken from Sakurai [see p. 412 (436) in the old (new) edition]. It is supposed to illustrate the concept of the scattering phase for scattering at a square-well potential. The figure is imprecise/incorrect in several places. Redraw the figure **carefully** yourselves, correcting the mistakes/imprecisions.

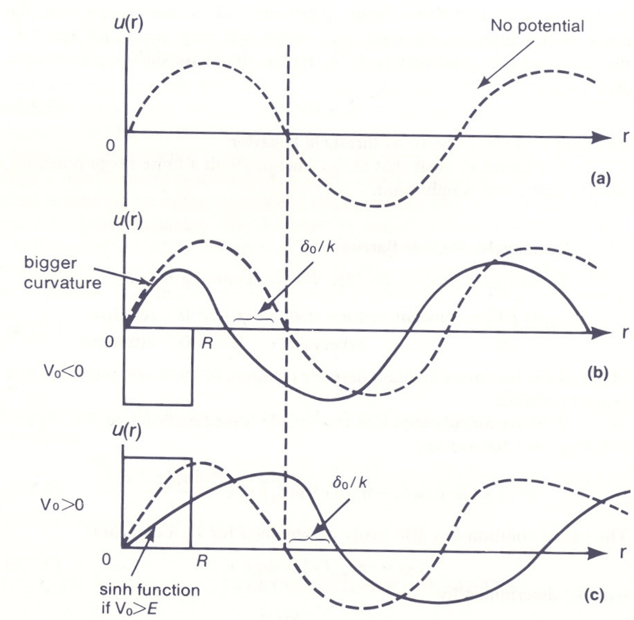

FIGURE 7.8. Plot of $u(r)$ versus r. (a) For $V = 0$ (dashed line). (b) For $V_0 < 0$, $\delta_0 > 0$ with the wave function (solid line) pulled in. (c) For $V_0 > 0$, $\delta_0 < 0$ with the wave function (solid line) pulled out.

(5) **Hard-sphere scattering** (5 Bonuspunkte)
Consider scattering of a particle of energy $E = \hbar^2 k^2/2m$ by the potential

$$V(r) = \begin{cases} \infty & r < a, \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine the scattering phase shifts δ_l, the partial scattering amplitudes f_l, and the partial (total) cross sections σ_l.
Hint: Use the ansatz $R_l(r) = \frac{1}{2}[e^{2i\delta_l} h_l(kr) + h_l^*(kr)]$ for the radial part of the wavefunction.

(b) Determine δ_0 and $R_0(r)$. Plot $R_0(r)$. Hint: $j_0(x) = \sin(x)/x$, $n_0(x) = -\cos(x)/x$.

(c) Consider the limit of low energies, $ka \ll 1$, and calculate the partial scattering amplitude f_0. For $l \neq 0$, show that $\lim_{ka\to 0} \sin\delta_l/k = 0$ and conclude that $f_l \to 0$. Discuss the differential and total cross section in the low-energy limit.
Hint: For $x \to 0$, $j_l(x) = x^l/1 \cdot 3 \cdot \ldots (2l+1)$, $n_l(x) = -1 \cdot 3 \cdot \ldots (2l-1)/x^{l+1}$

(d) * Show that in the limit of high energies, $ka \gg 1$, the total cross section is $\sigma_{\text{tot}} = 2\pi a^2$, i.e., twice the geometric cross section. Explanation?