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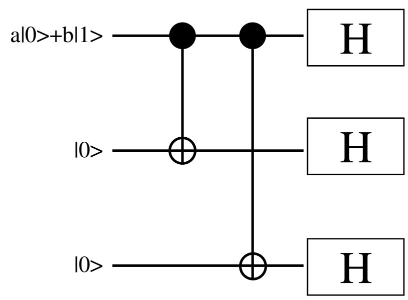
Nanophysics — Fall 2019

Quantum Computation and Quantum Communication
Exercise 2

due Friday, December 13, 2019

(1) **Phase-flip error correction code**
Consider the circuit shown below.



(a) Calculate the final three-qubit state.

(b) Express the state obtained in (a) in the $|+\rangle$ and $|-\rangle$ basis, defined by

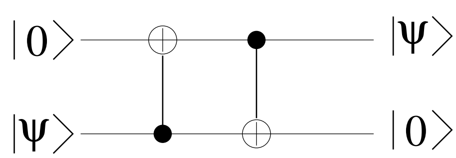
$$\hat{\sigma}_x|+\rangle = +|+\rangle \ ,$$
$$\hat{\sigma}_x|-\rangle = -|-\rangle \ .$$

(2) **Swap gate**
The SWAP gate is defined through the relation

$$\hat{U}_{\mathrm{SWAP}}|\chi\psi\rangle = |\psi\chi\rangle \ ,$$

i.e., the arbitrary states $|\chi\rangle$ and $|\psi\rangle$ of two qubits are exchanged (swapped).

(a) Show that the circuit below swaps the the first qubit in state $|0\rangle$ with the second qubit in an arbitrary state $|\psi\rangle$.



Note that this circuit only uses CNOT gates.

(b) The circuit shown in (a) requires $|\chi\rangle = |0\rangle$. Extend the circuit such that it implements the SWAP gate $\hat{U}_{\mathrm{SWAP}}$.

(3) **Square-root-of-SWAP gate**

The two-qubit SWAP gate is not sufficient for universal quantum computation. However, if the gate is pulsed for half a period, the resulting "square-root-of-SWAP" (or $\sqrt{\text{SWAP}} \equiv \hat{U}^{1/2}_{\text{SWAP}}$) becomes sufficient, because it allows one to implement an XOR-gate (up to some single-qubit rotations).

(a) Show that the matrix form of the $\hat{U}^{1/2}_{\text{SWAP}}$ takes the following form in the computational basis ($|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$):

$$\hat{U}^{1/2}_{\mathrm{SWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+i) & \frac{1}{2}(1-i) & 0 \\ 0 & \frac{1}{2}(1-i) & \frac{1}{2}(1+i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(b) Apply the $\sqrt{\text{SWAP}}$ gate to the input state $|\psi\rangle = |10\rangle$. What is the output state? Are the input and/or output states entangled?

(c) Repeat (b) using the states $|\psi\rangle = |00\rangle$, $|\psi\rangle = |01\rangle$, and $|\psi\rangle = |11\rangle$.