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Theorie der Supraleitung, Herbstsemester 2018

Blatt 6

Abgabe: 08.11.18, 12:00H (Treppenhaus 4. Stock)
Tutor: Alexandre Roulet Zi.: 4.10

(1) **The critical field H_{c3}** (10 Punkte)

In the course we studied the linearized Ginzburg-Landau equation for the case of an infinite superconductor in a homogeneous magnetic field $\mathbf{B} = (0, 0, B)$; vector potential e.g. $\mathbf{A} = (0, Bx, 0)$ and determined the critical field H_{c2} below which the GL equation has a nontrivial solution. In this problem we would like to do the same calculation for a superconducting half-space.
Hint: The boundary condition for ψ on the surface (normal vector $\mathbf{n}$) between a superconductor and an insulator (or vacuum) is $\mathbf{n} \cdot (-i\nabla - \frac{2\pi}{\Phi_0}\mathbf{A})\psi = 0$.

(a) Assume the superconductor fills the half-space $z < 0$, i.e., $\mathbf{n} = (0, 0, 1)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to?

(b) Now we consider the case that the field is parallel to the surface, i.e., the superconducting half-space $x < 0$ with $\mathbf{n} = (1, 0, 0)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to? Convince yourselves that by a clever choice of the position x_0 of the parabolic potentials you can obtain a lower eigenvalue of the linearized GL equation and therefore a critical field that is greater than H_{c2}.

(c) Find an estimate of the maximal critical field. E.g., use the variational principle, or solve the linearized GL numerically.

(2) **Critical current of a thin wire** (10 Extra-Punkte)

We consider the Ginzburg-Landau expression for the free energy of a superconductor,

$$F = F_n + \int d^3r \, \left(\alpha \left|\psi\right|^2 + \frac{\beta}{2} \left|\psi\right|^4 + \frac{1}{2m^*} \left| (\frac{\hbar}{i}\nabla - e^*\mathbf{A})\psi \right|^2 + \frac{h^2}{2\mu_0} \right) , \qquad (1)$$

with $\alpha < 0$, $\beta > 0$, $\psi(\mathbf{r}) = |\psi(r)| \, e^{i\varphi(r)}$, together with the expression

$$\mathbf{J}_s = e^* \left|\psi\right|^2 \mathbf{v}_s \qquad (2)$$

for the equilibrium supercurrent in the system, with $\mathbf{v}_s = (\hbar\nabla\varphi - e^*\mathbf{A})/m^*$.

(a) Show that, for boundary conditions which do not impose fields or gradients, one has $|\psi|^2 = |\psi_\infty|^2 = -\alpha/\beta$.
In the following, we consider a thin wire, so that we can neglect the term $h^2/(2\mu_0)$ in (1). We will also assume that the orientation of the wire with respect to any external field is such that $|\psi(r)|$ is spatially constant ($|\psi(r)| = |\psi|$).

(b) Use these hypotheses to simplify (1) into

$$F = F_n + \int d^3r \, \left(\alpha \left|\psi\right|^2 + \frac{\beta}{2} \left|\psi\right|^4 + \frac{m^*}{2} \left|\psi\right|^2 v_s^2 \right). \tag{3}$$

(c) Minimize the expression (3) for F to find the optimal value $|\psi|^{opt}$ for $|\psi|$ in terms of $|\psi_\infty|^2$, α and v_s.

(d) Give the expression of the supercurrent amplitude J_s^{opt} associated to $|\psi|^{opt}$.

(e) The critical current J_c of the wire can be calculated as the maximum of J_s^{opt} as a function of v_s. Show that

$$J_c = e^* \left|\psi_\infty\right|^2 \frac{2}{3} \left(\frac{2}{3} \frac{|\alpha|}{m^*} \right)^{1/2} . \tag{4}$$

Sketch the dependence of $|\psi|^{opt}$ and of J_s^{opt} as a function of v_s.

(f) Express J_c as a function of the critical field $H_c(T)$ of the superconductor and of the penetration depth $\lambda(T)$.

(g) Compare this critical current with the one that you would obtain with $\psi = \psi_\infty$ (that agrees with the so-called London theory) and explain the result.