

Theorie der Supraleitung, Herbstsemester 2018

Blatt 4

Abgabe: 25.10.18, 12:00H (Treppenhaus 4. Stock)

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(1) **Specific heat of a superconductor** (5 Punkte)

The elementary excitations with momentum \mathbf{k} of a superconductor have excitation energy $E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2(T)}$, where $\xi_{\mathbf{k}} = \epsilon_{\mathbf{k}} - \mu$ and $\Delta(T)$ is the temperature-dependent gap function. They are non-interacting fermions, i.e., their occupation number is given by the Fermi function $f_{\mathbf{k}} = (e^{\beta E_{\mathbf{k}}} + 1)^{-1}$, where $\beta = (k_B T)^{-1}$.

(a) The entropy of a system of non-interacting fermions is given by

$$S = -2k_B \sum_{\mathbf{k}} [(1 - f_{\mathbf{k}}) \ln(1 - f_{\mathbf{k}}) + f_{\mathbf{k}} \ln f_{\mathbf{k}}].$$

Show that the specific heat $c = \frac{1}{\text{Vol.}} T \frac{dS}{dT}$ can be written as

$$c = \frac{2}{\text{Vol.}} \beta k_B \sum_{\mathbf{k}} \left(-\frac{\partial f_{\mathbf{k}}}{\partial E_{\mathbf{k}}} \right) \left(E_{\mathbf{k}}^2 + \frac{1}{2} \beta \frac{d\Delta^2}{d\beta} \right). \quad (1)$$

Hint: $T \frac{d}{dT} = -\beta \frac{d}{d\beta}$.

Equation (1) is a bit tedious to obtain. If you do not manage, use the expression (1) and proceed with (b) and (c).

(b) Prove that in the limit $\Delta(T) \rightarrow 0$, i.e., for a normal metal, we obtain the well-known formula

$$c_n = \frac{2\pi^2}{3} N_n(0) k_B^2 T.$$

(c) The temperature dependence of Δ close to T_C is approximately given by

$$\Delta(T) = 3.06 k_B \sqrt{T_C} \sqrt{T_C - T} \quad \text{for } T \lesssim T_C,$$

and (of course) $\Delta(T) = 0$ for $T > T_C$.

Calculate the jump $\frac{c - c_n}{c_n}$ in the specific heat at $T = T_C$.

(d) What is the temperature dependence of c for low temperatures, $k_B T / \Delta(0) \rightarrow 0$?

(2) **Transition probabilities and coherence effects** (5 Punkte)

Read and understand the Section *Calculation of Transition Probabilities* in de Gennes' book and explain in your own words why the (ultra-)sound attenuation in a superconductor is lowered below T_C whereas the NMR relaxation rate is increased.