Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 207 36 92, Christoph.Bruder@unibas.ch)

Theory of Superconductivity, Frühjahrsemester 2026

Blatt 9

Abgabe: 21.05.26, 12:00H (auf adam oder Treppenhaus 4. Stock)
Tutor: Tobias Nadolny Zi.: 4.48

(1) **Clogston-Chandrasekhar-Pauli limit**
At the beginning of the course we discussed the thermodynamic critical field H_C of a Type-I superconductor. It is due to orbital depairing and is obtained by comparing the condensation energy with the energy of the expelled field (both per unit volume).

(a) Show that this argument leads to $H_C = \Delta\sqrt{\frac{N(0)}{\mu_0}}$.

(b) If the field is applied parallel to a thin superconducting film, orbital depairing is suppressed (why?). The limiting field H_C^P is determined by paramagnetic depairing (the field tries to align the spins and will eventually break singlet Cooper pairs). Use the expression $-\frac{1}{2}\chi_{\mathrm{Pauli}}\mu_0 H^2$ for the paramagnetic energy gain in a normal metal to derive the limiting field H_C^P at temperature $T = 0$:

$$\mu_0 H_C^P = \frac{\Delta(0)}{\sqrt{2}\mu_B} = 1.86 T_C \,[\mathrm{Tesla/Kelvin}]\,.$$

How does it compare to the thermodynamic critical field H_C?

(2) **Quasiparticle spectrum for a state of uniform current flow**
A state of uniform current flow is described by a pair potential of the form $\Delta(\mathbf{r}) = |\Delta|\exp(2i\mathbf{qr})$ where $\mathbf{q}$ is a vector in the direction of flow (the average momentum per electron in this state is $\hbar\mathbf{q}$).
Solve the Bogoliubov-de Gennes equations for this situation and show that for $|\mathbf{q}| \ll k_{\mathrm{F}}$, the quasiparticle spectrum is given by

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + |\Delta|^2} + \frac{\hbar^2}{m}\mathbf{k}\cdot\mathbf{q}\,.$$

Discuss the resulting energy gap.
Hint: assume that $|\Delta|$ is constant, i.e., neglect self-consistency.