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Theory of Superconductivity, Frühjahrsemester 2026

Blatt 8

Abgabe: 07.05.26, 12:00H (auf adam oder Treppenhaus 4. Stock)
Tutor: Bethany Davies Zi.: 4.10

(1) **Cooper-pair box**
Consider the Cooper pair box Hamiltonian

$$\hat{H}_{\mathrm{CPB}} = 4E_C(\hat{N} - n_g)^2 - E_J \cos(\hat{\varphi})$$

where $n_g = -q/(2e) = -C_g V_g/(2e)$.

(a) Show that $[\hat{N}, \hat{\varphi}] = i$ leads to $[\hat{N}, e^{i\hat{\varphi}}] = -e^{i\hat{\varphi}}$. Use this commutator to express $\cos(\hat{\varphi})$ and $\hat{H}_{\mathrm{CPB}}$ in the eigenbasis $\{|N\rangle\}_{N\in\mathbb{Z}}$ of the charge operator.

(b) For $E_c \gg E_J$, sketch the energy spectrum as a function of n_g.

(c) Consider the point $n_g = N - 1/2$ for an integer $N \in \mathbb{Z}$. Estimate the energy splitting of the two lowest eigenstates.

(d) Solve the Schrödinger equation of the Cooper-pair box

$$E\psi(N) = H\psi(N) = 4E_C\left(N - n_g\right)^2 \psi(N) - \frac{E_J}{2}\left[\psi(N-1) + \psi(N+1)\right]$$

numerically. Plot the eigenvalues as a function of n_g for different values of the ratio E_C/E_J and interpret your result. Compare with (b) and (c).