

Theory of Superconductivity, Frühjahrssemester 2026

Blatt 7

Abgabe: 23.04.26, 12:00H (auf adam oder Treppenhaus 4. Stock)

Tutor: Bethany Davies, Zi.: 4.10

(1) Resistively shunted Josephson junction

In the course we studied the RCSJ model. In the overdamped case $RC\omega_p \ll 1$ where $\omega_p^2 = 2eI_{c0}/(\hbar C)$, its equation of motion reads

$$\frac{d\phi}{dt} = \frac{2eI_{c0}R}{\hbar} \left(\frac{I}{I_{c0}} - \sin\phi \right). \quad (1)$$

Here, ϕ is the phase difference across the junction, I_{c0} the critical current, and R the shunt resistance.

- (a) Solve this equation for $I < I_{c0}$ and interpret the solution.
- (b) We now consider the case $I > I_{c0}$. Show that there is no stationary solution. Determine the time period T of the solution and use the (time-averaged) second Josephson relation $2eV_{av}/\hbar = 2\pi/T$ to find the time-averaged voltage V_{av} across the junction. Plot the IV -characteristics obtained from (a) and (b).
- (c) Integrate (1) numerically for $I \gtrsim I_{c0}$ (or solve it exactly, which is possible). Plot and discuss the behavior of $\phi(t)$ and $V(t)$ for different currents.