

Theory of Superconductivity, Frühjahrsemester 2026

Blatt 6

Abgabe: 16.04.26, 12:00H (auf adam oder Treppenhaus 4. Stock)

Tutor: Bethany Davies, Zi.: 4.48

(1) **The critical field H_{c3}** (10 Punkte)

In the course we studied the linearized Ginzburg-Landau equation for the case of an infinite superconductor in a homogeneous magnetic field $\mathbf{B} = (0, 0, B)$; vector potential e.g. $\mathbf{A} = (0, Bx, 0)$, and determined the critical field H_{c2} below which the GL equation has a nontrivial solution. In this problem we would like to do the same calculation for a superconducting half-space.

Hint: The boundary condition for ψ on the surface (normal vector \mathbf{n}) between a superconductor and an insulator (or vacuum) is $\mathbf{n} \cdot (-i\nabla - \frac{2\pi}{\Phi_0}\mathbf{A})\psi = 0$.

- (a) Assume the superconductor fills the half-space $z < 0$, i.e., $\mathbf{n} = (0, 0, 1)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to?
- (b) Now we consider the case that the field is parallel to the surface, i.e., the superconducting half-space $x < 0$ with $\mathbf{n} = (1, 0, 0)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to? Convince yourselves that by a clever choice of the position x_0 of the parabolic potentials you can obtain a lower eigenvalue of the linearized GL equation and therefore a critical field that is greater than H_{c2} .
- (c) Find an estimate of H_{c3} , the maximal critical field. E.g., use the variational principle, or solve the linearized GL numerically.