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Theory of Superconductivity, Frühjahrsemester 2026

Blatt 1: Self-study

The purpose of problems 1 and 2 is to review the basic properties of the ideal Fermi and Bose gases. This is a reading assignment; use any textbook or other source.

(1) **Free (= ideal) Fermi gas**
Use a textbook of your choice (e.g., S.H. Simon, The Oxford solid state basics, pages 27 – 32) to review the basic properties of the free Fermi gas. In particular, you should understand and remember

(a) the meaning and the shape of the Fermi function $f(E) = \dfrac{1}{\exp((E-\mu)/k_\mathrm{B}T)+1}$.

(b) the meaning and typical numerical values of the Fermi energy ϵ_F, the Fermi wavevector k_F, the Fermi velocity v_F, the Fermi temperature $T_\mathrm{F} = \epsilon_\mathrm{F}/k_\mathrm{B}$.

(c) the relation $k_\mathrm{F}^3 = 3\pi^2 n$ between the Fermi wavevector and the density $n = N/V$ of the fermions.

(d) the meaning of the density of states (per unit volume; for one spin direction) $N(E) = \dfrac{3}{4}\dfrac{n}{\epsilon_\mathrm{F}}\left(\dfrac{E}{\epsilon_\mathrm{F}}\right)^{1/2}$ and its value at the Fermi energy $N(\epsilon_\mathrm{F}) = \dfrac{mk_\mathrm{F}}{2\pi^2\hbar^2}$.

(e) the specific heat C at low temperatures $T \ll T_\mathrm{F}$, $C = \gamma T$ where $\gamma = \dfrac{2\pi^2}{3}k_\mathrm{B}^2 N(\epsilon_\mathrm{F})$.

(f) Pauli (spin) paramagnetism of the free electron gas, $\chi_\mathrm{Pauli} = \mu_0 \dfrac{\mathrm{d}M}{\mathrm{d}B} = 2\mu_0\mu_\mathrm{B}^2 N(\epsilon_\mathrm{F})$ where $\mu_\mathrm{B} = |e|\hbar/(2m)$ is the Bohr magneton. The orbital motion of the electrons leads to Landau diamagnetism, $\chi_\mathrm{Landau} = -\dfrac{1}{3}\chi_\mathrm{Pauli}$. The typical numerical values of χ_Pauli and χ_Landau are of order 10^{-6}.

(2) **Free (= ideal) Bose gas**
Use a textbook of your choice (e.g., Tilley[2], Superfluidity and Superconductivity, pages 31 – 38) to review the basic properties of the free Bose gas. In particular, you should understand and remember

(a) the meaning and the shape of the Bose function $f_\mathrm{B}(E) = \dfrac{1}{\exp((E-\mu)/k_\mathrm{B}T)-1}$.

(b) Bose-Einstein condensation: the lowest one-particle state will be occupied by a macroscopic number $N_0(T)$ of particles at temperatures $T < T_\mathrm{B} = \dfrac{2\pi\hbar^2}{mk_\mathrm{B}}\left(\dfrac{N}{\zeta(3/2)V}\right)^{2/3} = 3.31\dfrac{\hbar^2}{mk_\mathrm{B}}n^{2/3}$. In fact, $\dfrac{N_0(T)}{N} = 1 - \left(\dfrac{T}{T_\mathrm{B}}\right)^{3/2}$.