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Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2025

Blatt 0

Besprechung: 21.02.2025; 13:15 – 15:00 (wird in der Übung gerechnet, keine Abgabe!)
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(1) **Free fall of a ball: numerically solving ordinary differential equations (ODEs)**

Consider a ball (radius 0, we are physicists...) in the gravitational field of the Earth. If we assume the field to be homogeneous, the differential equation which describes the free fall of the ball reads

$$m\frac{\mathrm{d}^2 h}{\mathrm{d}t^2} = -mg \,. \tag{1}$$

Here, $h(t)$ denotes its height, m its mass, and g is the gravitational acceleration.

We assume that at time $t = t_0 = 0$, the initial value of the height h is $h(t_0) = h_0$, and the initial value of the velocity $v(t) \equiv \dot{h}(t) \equiv \frac{\mathrm{d}h}{\mathrm{d}t}(t)$ is $v(t_0) = v_0$.

In the following, we will describe a numerical procedure that can be applied to solve arbitrary ODEs for which the initial values are given.

(a) Equation (1) is a second-order ODE. Show that any explicit one-dimensional ODE of order n can be reduced to n differential equations of order 1,

$$\dot{\mathbf{u}}(t) = \mathbf{f}(t, \mathbf{u}) \,, \qquad \mathbf{u}(t_0) = \mathbf{u}_0 \,, \tag{2}$$

here, $\mathbf{u}$, $\mathbf{f}$, and $\mathbf{u}_0$ are n-dimensional vectors.
Do this explicitly for Eq. (1). Solution:

$$\mathbf{u}(t) = \begin{pmatrix} h(t) \\ v(t) \end{pmatrix} \; ; \qquad \mathbf{f}(t, \mathbf{u}) = \begin{pmatrix} v(t) \\ -g \end{pmatrix} \; ; \qquad \mathbf{u}_0 = \begin{pmatrix} h_0 \\ v_0 \end{pmatrix} \,. \tag{3}$$

(b) The explicit Euler scheme

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \Delta t \, \mathbf{f}(t_k, \mathbf{u}_k) \tag{4}$$

gives an approximate solution $\mathbf{u}(t_j) = \mathbf{u}_j$ to the initial value problem defined by Eq. (2) in the interval $[t_0, T]$. The t_j, $j = 0, 1, \ldots, N$ are a discretization of the time interval; e.g., $t_j = t_0 + j \cdot \Delta t$ and $\Delta t = (T - t_0)/N$.
Assuming $t_0 = 0$ and the initial values $h_0 = 50$ m, $v_0 = 0$, solve the differential equation (3) numerically by applying the explicit Euler scheme for $T = 10$ s and $N = 100$.

(c) Compare with the exact solution of (1)

$$h(t) = h_0 + v_0 t - \frac{1}{2} g t^2 \,. \tag{5}$$

Plot both the approximate and the exact solution.

(d) Repeat (b) and (c) using Julia's package for solving ordinary differential equations **OrdinaryDiffEq**. Use the skeleton program provided on ADAM if necessary.