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Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2024

Blatt 9

Abgabe: 16.05.2024, 12:00
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(1) **Wigner and Husimi-Q distributions** (3 Punkte)

(a) Plot the Wigner (quasiprobability) distribution distribution for both the Fock states $|0\rangle$, $|1\rangle$, $|2\rangle$ and the coherent states $|\alpha = 0\rangle$, $|\alpha = 1\rangle$, $|\alpha = 2\rangle$. Reproduce Figs. 8.12, 8.13 of Eichler and Zilberberg.

(b) Repeat (a) for the Husimi-Q distribution and reproduce Figs. 8.10, 8.11 of Eichler and Zilberberg.

(c) Plot the Wigner distribution both for a mixed state and a superposition state containing $|\alpha = +1\rangle$ and $|\alpha = -1\rangle$ with equal weight. What is the difference?

(2) **Damped quantum harmonic oscillator** (3 Punkte)
Consider the following quantum master equation of a damped harmonic oscillator:

$$\frac{d}{dt}\rho = -i[H, \rho] + \gamma\mathcal{D}[a]\rho \,, \tag{1}$$

where $a^\dagger$, a are the raising and lowering operators of the oscillator, $H = \omega_0 a^\dagger a$ (we neglect the zero-point energy and set $\hbar = 1$), and $\mathcal{D}[O]\rho = O\rho O^\dagger - \frac{1}{2}\{O^\dagger O, \rho\}$ is the definition of the Lindblad dissipator for any operator O.

(a) Calculate the matrix elements $\langle n| \dots |m\rangle$ of Eq. (1).

(b) Choose $|\psi(t = 0)\rangle = |1\rangle$ as initial state and solve the equation obtained in (a). Interpretation?

(c) Compute the differential equation governing the time evolution for the expectation value $\langle a\rangle = \mathrm{Tr}[a\rho]$. Bring the differential equation into the form of a classical damped harmonic oscillator $\ddot{x} + \nu\dot{x} + \omega x = 0$. What are ν and ω in terms of γ and ω_0?

(3) **Quantum van der Pol oscillator** (4 Punkte + 2 Bonuspunkte)
The quantum van der Pol oscillator is defined by the quantum master equation

$$\frac{d}{dt}\rho = -i[\omega_0 a^\dagger a, \rho] + \gamma_1\mathcal{D}[a^\dagger]\rho + \gamma_2\mathcal{D}[a^2]\rho \,. \tag{2}$$

(a) Interpret the terms proportional to γ_1 and γ_2.

(b) Compute the differential equation governing the time evolution for the expectation value $\alpha \equiv \langle a \rangle = \mathrm{Tr}[a\rho]$.

(c) Approximate the term $\langle a^\dagger a^2 \rangle \approx |\alpha|^2 \alpha$ to obtain a closed differential equation for α. When is this approximation valid?

(d) Set $\alpha = re^{i\phi}$ and show that there is a limit cycle.

(e) Bonus: Use a package like QuantumOptics.jl to compute and visualize the time evolution of Eq. (2) starting from a coherent state. Set $\omega_0/\gamma_1 = 1$ and test different values of γ_2/γ_1 (see for instance the function QuantumOptics.timeevolution.master and the example at
`https://docs.qojulia.org/examples/pumped-cavity/#Pumped-cavity`,
where you can ignore the Monte Carlo trajectories). Discuss the time evolution.