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Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2024

Blatt 5

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(1) **Fixed points in Hamiltonian systems** (2 Punkte)
Consider an autonomous Hamiltonian system

$$\dot{q} = \frac{\partial H(p,q)}{\partial p}; \qquad \dot{p} = -\frac{\partial H(p,q)}{\partial q}$$

Show that fixed points are either saddle points or centers.

(2) **Homoclinic/infinite-period bifurcations in a Josephson junction** (4 Punkte)
Read and understand Section 8.5 of Strogatz. Note that in the subsection *Homoclinic bifurcation* as well as in the paragraph around Fig. 8.5.11, I_c should be replaced by I_r (for *retrapping current*) since it is different from the critical current in Eq. (1) of Sec. 8.5.

(a) Reproduce Fig. 8.5.11

(3) **Trapping region for the Lorenz equations** (2 Punkte)
Show that there is a certain ellipsoidal region E of the form

$$rx^2 + \sigma y^2 + \sigma(z - 2r)^2 \leq C$$

such that all trajectories of the Lorenz equations eventually enter E and stay there forever.
Bonus problem: Try to obtain the smallest possible value of C with this property.

(4) **Numerical solution of the Lorenz equations** (2 Punkte + 2 Bonuspunkte)
Solve the Lorenz equations numerically.

(a) Choose the "standard" parameter values $\sigma = 10$, $b = 8/3$, $r = 28$ and plot z over x, x over t, and z over x and y for a typical initial condition, e.g., $\mathbf{x}(0) = (1, 5, 10)$.

(b) Vary r and confirm and discuss the various behaviors described in Section 9.5 of Strogatz.

(5) **Lorenz map** (3 Bonuspunkte)
Read the first two pages of Section 9.4 in Strogatz.
Compute the Lorenz map: use a computer to reproduce Fig. 9.4.3.