Departement Physik, Universität Basel
Prof. C. Bruder (Zimmer 4.2, Tel.: 061 207 36 92, Christoph.Bruder@unibas.ch)

Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2024

Blatt 4

Abgabe: 04.04.2024
Tutor: Marcelo Janovitch Zi. 515 Phys. Chem.; m.janovitch@unibas.ch

---

(1) **Conservative systems** (2 Punkte)
Given a system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, a *conserved quantity* is a real-valued continuous function $E(\mathbf{x})$ that is constant on trajectories, i.e., $dE/dt = 0$.
To avoid trivial examples, we also require that $E(\mathbf{x})$ be nonconstant on every open set.
Show that a conservative system (i.e., a system for which a conserved quantity exists) cannot have any attractive fixed points.

(2) **Liapunov function** (3 Punkte)
Show that the system $\dot{x} = y - x^3$, $\dot{y} = -x - y^3$ has no closed orbits by constructing a Liapunov function $V = ax^2 + by^2$ with suitable a, b.

(3) **Absence of closed orbits** (2 Punkte)
Consider $\dot{x} = x^2 - y - 1$, $\dot{y} = y(x - 2)$.

- (a) Show that there are three fixed points and classify them.
- (b) By considering the three straight lines through pairs of fixed points, show that there are no closed orbits.
- (c) Plot the phase portrait.

(4) **Index theory** (3 Punkte)
Read and understand Section 6.8 in Strogatz until Theorem 6.8.2.

- (a) Show that each of the following fixed points has an index equal to $+1$:
  (i) stable spiral (ii) unstable spiral (iii) center (iv) star (v) degenerate node.
- (b) Use index theory to show that the system $\dot{x} = x(4 - y - x^2)$, $\dot{y} = y(x - 1)$ has no closed orbits.