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Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2024

Blatt 3

Abgabe: 21.03.2024
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(1) **Attracting/Liapunov stable/asymptotically stable** (2 Punkte)
For each of the following systems, decide whether the origin is attracting, Liapunov stable, asymptotically stable (= attracting *and* Liapunov stable), or none of the above.

(a) $\dot{x} = 2y$, $\dot{y} = x$

(b) $\dot{x} = 0$, $\dot{y} = x$

(c) $\dot{x} = 0$, $\dot{y} = -y$

(d) $\dot{x} = -x$, $\dot{y} = -5y$

(2) **Classification of fixed points** (2 Punkte)
Plot the phase portrait and classify the fixed points in the following linear systems. If the eigenvectors are real, indicate them in your sketch.

(a) $\dot{x} = 5x + 10y$, $\dot{y} = -x - y$

(b) $\dot{x} = 3x - 4y$, $\dot{y} = x - y$

(c) $\dot{x} = -3x + 2y$, $\dot{y} = x - 2y$

(d) $\dot{x} = 5x + 2y$, $\dot{y} = -17x - 5y$

(3) **Intersection of trajectories?** (2 Punkte)
We claimed that different trajectories can never intersect. But in many phase portraits, different trajectories appear to intersect at a fixed point. Is there a contradiction here?

(4) **A linear center that's actually a nonlinear spiral** (2 Punkte)
Consider the system

$$\dot{x} = -y - x^3, \qquad \dot{y} = x \ .$$

Show that the origin is a spiral, although the linearization predicts a center.

(5) **Saddle switching and structural stability** (2 Punkte)
Consider the system

$$\dot{x} = a + x^2 - xy, \qquad \dot{y} = y^2 - x^2 - 1 \ ,$$

where a is a parameter.

(a) Sketch the phase portrait for $a = 0$. Show that there is a trajectory connecting two saddle points (a *saddle connection*).

(b) Use a computer to sketch the phase portrait for $a < 0$ and $a > 0$.

(6) **Bonus problems** (4 Bonuspunkte)
In Strogatz, there are many fun problems in areas like economy or or politics; see e.g. problem 6.4.9 (Model of a national economy)
or
problem 6.4.11 (Leftists, rightists, centrists). Solve one of those and present it in the exercise class.