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Classical and Quantum Nonlinear Dynamics
Frühjahrsemester 2024

Blatt 10

Abgabe: 23.05.2024, 12:00
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(1) **Phase locking of a quantum van der Pol oscillator** (4 Punkte)
In Blatt 9, problem (3), we encountered the quantum van der Pol (qvdP) oscillator. We now study phase locking of a qvdP oscillator subject to an external drive. The quantum master equation is

$$\frac{d}{dt}\rho = -i[H, \rho] + \gamma_1 \mathcal{D}[a^\dagger]\rho + \gamma_2 \mathcal{D}[a^2]\rho \, ,$$

where the Hamiltonian is $H = -\Delta a^\dagger a + i\Omega(a - a^\dagger)$ (in the rotating frame of the drive). For simplicity, we have set $\hbar = 1$.

(a) Download the paper Walter et al., PRL **112**, 094102 (2014). Read the first 1.75 pages until *Spectra and observed frequency.* How can phase locking be observed in the quantum van der Pol oscillator? How does it depend on detuning and drive strength?

(b) Reproduce the plots of the Wigner distribution in Fig. 1 and 2 (a-d). If necessary, use the code in the file 'Blatt_10_1.jl' (on adam).

(2) **Master equation** (3 Punkte)
Starting from the Hamiltonian H for the Kerr parametric oscillator (Eq. (10.3) of Eichler/Zilberberg), derive the Hamiltonian $\tilde{H}$ in the rotating frame shown in Eq. (10.4) Then, derive the equation of motion for the expectation value $\tilde{\alpha} \equiv \langle \tilde{a} \rangle$ (generalized case, Eq. (10.8) of Eichler/Zilberberg). To do that, you have to use the semiclassical approximation, i.e., factorize expectation values of products like $\langle (\tilde{a}^\dagger)^n (\tilde{a})^m \rangle \approx (\langle \tilde{a} \rangle^*)^n \langle \tilde{a} \rangle^m$. Write down all the steps of the derivation.

(3) **Meissner equation** (3 Punkte + 2 Bonuspunkte)
The Meissner equation

$$\ddot{x} + \omega_0^2 [1 - \lambda \psi_M(t)] x + \Gamma \dot{x} = 0 \tag{1}$$

is a paradigmatic example of a parametrically driven system. Here, $\psi_M(t)$ is a periodic stepwise constant function with period $T_p = 2\pi/\omega_p$: $\psi_M(t) = -1$ for $0 < t < T_p/2$ and $\psi_M(t) = 1$ for $T_p/2 < t < T_p$. In certain regions of the λ-ω_p-diagram, the system is unstable and the amplitude will tend to infinity.

(a) Solve Eq. (1) numerically and reproduce Fig. 1.6 in Eichler/Zilberberg.

(b) Bonus points: Read and understand p. 14 - 16 in Eichler/Zilberberg and fill in the missing steps.