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Theory of Superconductivity, Frühjahrsemester 2023

Blatt 7

Abgabe: 27.04.23, 12:00H (Treppenhaus 4. Stock)
Tutor: Tobias Nadolny, Zi.: 4.48

(1) **The critical field H_{c3}** (10 Punkte)

In the course we studied the linearized Ginzburg-Landau equation for the case of an infinite superconductor in a homogeneous magnetic field $\mathbf{B} = (0, 0, B)$; vector potential e.g. $\mathbf{A} = (0, Bx, 0)$, and determined the critical field H_{c2} below which the GL equation has a nontrivial solution. In this problem we would like to do the same calculation for a superconducting half-space.
Hint: The boundary condition for ψ on the surface (normal vector $\mathbf{n}$) between a superconductor and an insulator (or vacuum) is $\mathbf{n} \cdot (-i\nabla - \frac{2\pi}{\Phi_0}\mathbf{A})\psi = 0$.

(a) Assume the superconductor fills the half-space $z < 0$, i.e., $\mathbf{n} = (0, 0, 1)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to?

(b) Now we consider the case that the field is parallel to the surface, i.e., the superconducting half-space $x < 0$ with $\mathbf{n} = (1, 0, 0)$. What are the solutions of the linearized GL equation discussed in the course that fulfill the boundary condition? Which critical field do they correspond to? Convince yourselves that by a clever choice of the position x_0 of the parabolic potentials you can obtain a lower eigenvalue of the linearized GL equation and therefore a critical field that is greater than H_{c2}.

(c) Find an estimate of H_{c3}, the maximal critical field. E.g., use the variational principle, or solve the linearized GL numerically.