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Theory of Superconductivity, Frühjahrsemester 2023

Blatt 3

Abgabe: 23.3.23, 12:00H (Treppenhaus 4. Stock)
Tutor: Tobias Kehrer, Zi.: 4.48

(1) **Temperature dependence of Δ** (6 Punkte)

The temperature dependence of $\Delta(T)$ is determined by the gap equation

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{l}} V_{\mathbf{kl}} \frac{\Delta_{\mathbf{l}}}{2E_{\mathbf{l}}} \tanh(\frac{\beta E_{\mathbf{l}}}{2}) \qquad (1)$$

where $E_{\mathbf{l}} = \sqrt{\xi_{\mathbf{l}}^2 + \Delta_{\mathbf{l}}^2}$ and $\beta = (k_B T)^{-1}$.

(a) Assuming that $V_{\mathbf{kl}} = -V$ if $|\xi_{\mathbf{k}}|$, $|\xi_{\mathbf{l}}| < \hbar\omega_c$ and 0 otherwise, show that the gap equation can be written as

$$1 = VN(0) \int_0^{\hbar\omega_c} \mathrm{d}\xi \frac{1}{E} \tanh(\frac{\beta}{2} E) \qquad (2)$$

where $E = \sqrt{\xi^2 + \Delta^2}$. Use (2) at $T = T_C$ to eliminate $VN(0)$ in favor of T_C and solve the resulting equation numerically in the temperature interval $[0, 2T_C]$.

(b) Study the behavior of $\Delta(T)$ for $T \to T_C$ and for $T \to 0$, either analytically or numerically.

(2) **Superconducting density of states** (4 Punkte)

The excitation energy of the quasi-particle is given by

$$E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^2 + \Delta^2},$$

where $\xi_{\mathbf{k}}$ is the independent-particle kinetic energy relative to the Fermi energy.
The number of excitations in a superconductor in the energy interval $[E, E + dE]$ is $N_s(E)dE$, here, $N_s(E)$ is the density of states of the superconductor. The corresponding number in the normal state is $N_n(\xi)d\xi \approx N_n(0)d\xi$, here, $N_n(\xi)$ is the density of states of the normal metal. Calculate $N_s(E)/N_n(0)$ by equating the two expressions.
Compare the superconducting density of states with the normal one and discuss the origin of the divergence of $N_s(E)$ at $E = \Delta$.

(3) **Ground-state energy** (good practice...5 Zusatz-Punkte)

Define $\langle E \rangle = \langle \psi_G | H - \mu \hat{N} | \psi_G \rangle$. Follow the steps in Tinkham 3.4.2 to evaluate the difference between the ground-state energy of the BCS state and the free Fermi gas.
Result:

$$\langle E \rangle_s - \langle E \rangle_n = -\frac{1}{2} N(0) \Delta^2$$