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# Theory of Superconductivity, Früjahrsemester 2023

**Blatt 10**

Abgabe: 25.05.23, 12:00H (Treppenhaus 4. Stock)
Tutor: Julian Arnold Zi.: 4.10

(1) **Cooper-pair box** (10 Punkte)

Consider the Cooper pair box Hamiltonian

$$\hat{H}_{\text{CPB}} = 4E_C(\hat{N} - n_g)^2 - E_J \cos(\hat{\varphi})$$

where $n_g = -q/(2e) = -C_g V_g/(2e)$.

(a) Show that $[\hat{N}, \hat{\varphi}]_- = i$ leads to $[\hat{N}, e^{i\hat{\varphi}}]_- = -e^{i\hat{\varphi}}$. Use this commutator to express $\cos(\hat{\varphi})$ and $\hat{H}_{\text{CPB}}$ in the eigenbasis $\{|N\rangle\}_{N\in\mathbb{Z}}$ of the charge operator.

(b) For $E_c \gg E_J$, sketch the energy spectrum as a function of $n_g$.

(c) Consider the point $n_g = N - 1/2$ for an integer $N \in \mathbb{Z}$. Estimate the energy splitting of the two lowest eigenstates.

(d) Solve the Schrödinger equation of the Cooper-pair box

$$E\psi(N) = H\psi(N) = 4E_C\left(N - n_g\right)^2\psi(N) - \frac{E_J}{2}\left[\psi(N-1) + \psi(N+1)\right]$$

numerically. Plot the eigenvalues as a function of $n_g$ for different values of the ratio $E_C/E_J$ and interpret your result. Compare with (b) and (c).

(2) **Zusatzaufgabe: Shapiro steps** (10 Extra-Punkte)

Assume an ideal Josephson junction that is voltage-biased with

$$V = V_0 + V_1 \cos\omega_1 t\,.$$

(a) Calculate the phase difference $\gamma(t)$ across the junction and write down the current $I = I_c \sin\gamma$.

(b) Use the addition theorem and the Fourier expansions of $\sin(z\sin(\theta))$ and $\cos(z\sin(\theta))$ (look them up!) to express the current $I$ as a trigonometric series.

(c) Find the values of $V_0$ for which the current has a DC component.

(d) Since the integration constant in $\gamma(t)$ is arbitrary, the DC current corresponding to one of the "Shapiro" values in (c) can take different values. Find the width of the corresponding current region. These plateaus are called "Shapiro steps".