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Advanced quantum mechanics and quantum field theory, FS 2021

Blatt 4

Submission: 8.04.2021, 12:00H, **on adam in the appropriate folder.**
One file per submission please; the filename HAS TO contain your name, or the submission will not be corrected!
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(1) **Noether currents and charges for Lorentz transformations** (3 Punkte)
Read Example 10.4 on p. 95 - 96 in Lancaster/Blundell carefully.
Derive Eqs. (10.32) and (10.33) and show how (10.34) follows. Explain the meaning of the components of the Noether currents (10.38) and (10.39). Explain Eq. (10.40) in your own words.

(2) **Energy-momentum tensor of the electromagnetic field** (4 Punkte)

(a) The Lagrange density of the electromagnetic field is

$$\mathcal{L}(A^\rho, \partial^\sigma A^\rho) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \, .$$

Show that the momentum density is given by $\Pi^{\sigma\rho} = -F^{\sigma\rho}$.
(Why is it a tensor in this case and not a vector?)

(b) Show that the energy-momentum tensor can be written as

$$T^{\mu\nu} = -F^{\mu\sigma}\partial^\nu A_\sigma + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \, .$$

Show that adding $\partial_\lambda X^{\lambda\mu\nu}$ to $T^{\mu\nu}$ where $X^{\lambda\mu\nu} = -X^{\mu\lambda\nu}$ does not change the conservation law $\partial_\mu T^{\mu\nu} = 0$ and hence does not change the conserved charges.

(c) Show that $T^{\mu\nu}$ (that is not symmetric) can be symmetrized by adding $\partial_\lambda X^{\lambda\mu\nu}$ where $X^{\lambda\mu\nu} = F^{\mu\lambda} A^\nu$ (which obviously fulfills $X^{\lambda\mu\nu} = -X^{\mu\lambda\nu}$!)
Show that $\tilde{T}^{\mu\nu} = T^{\mu\nu} + \partial_\lambda X^{\lambda\mu\nu}$ takes the form

$$\tilde{T}^{\mu\nu} = F^{\mu\sigma} F_\sigma{}^\nu + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

and convince yourselves that it is indeed symmetric.

(d) Show that $\tilde{T}^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2)$ and $\tilde{T}^{0i} = (\mathbf{E} \times \mathbf{B})^i$. Interpretation?

(3) **Noether current and charge for a theory with an internal symmetry** (3 Punkte)

Consider the theory described by

$$\mathcal{L} = \frac{1}{2}[\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2] - \frac{1}{2} m^2 (\phi_1^2 + \phi_2^2) - g(\phi_1^2 + \phi_2^2)^2$$

where ϕ_1, ϕ_2 are two scalar fields. As discussed in class, it has an internal $SO(2)$ symmetry, i.e., is invariant under rotations in the $\phi_1 - \phi_2$ plane.
Find the corresponding Noether current and charge and interpret them.