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Advanced quantum mechanics and quantum field theory, FS 2021

Blatt 3

Submission: 25.03.2021, 12:00H, **on adam in the appropriate folder.**
One file per submission please; the filename HAS TO contain your name, or the submission will not be corrected!
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(1) **"The death of single-particle quantum mechanics"** (3 Punkte)
Read Lancaster/Blundell p. 75 - 76 carefully.
Are you convinced by their argument? If so, how do you justify to use (8.16) together with the Klein-Gordon equation which has a second-order time derivative.
Would you change anything in the notation of Example 8.1?

(2) **Infinitesimal Lorentz transformations** (7 Punkte)

(a) Show that the generator for a Lorentz boost in x-direction is

$$K^1 = \frac{1}{i}\left.\frac{\partial D(\phi^1)}{\partial \phi^1}\right|_{\phi^1=0} = -i\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Here, $\tanh\phi^1 = \beta^1$ is the rapidity – explain the difference to the velocity. Do the same for boosts in the y-direction and z-direction.

(b) Conclude that a general infinitesimal boost parametrized by the velocities β^i along the x^i-axis is given by the Lorentz transformation

$$\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & \beta^1 & \beta^2 & \beta^3 \\ \beta^1 & 1 & 0 & 0 \\ \beta^2 & 0 & 1 & 0 \\ \beta^3 & 0 & 0 & 1 \end{pmatrix}.$$

(c) Show also that an infinitesimal rotation parametrized by the rotation angles θ^i about the x^i-axis is given by the Lorentz transformation

$$\Lambda^\mu{}_\nu = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \theta^3 & -\theta^2 \\ 0 & -\theta^3 & 1 & \theta^1 \\ 0 & \theta^2 & -\theta^1 & 1 \end{pmatrix}.$$

(d) Hence show that a general infinitesimal Lorentz transformation can be written as $\mathbf{\Lambda} = 1 + \boldsymbol{\omega}$ where

$$\omega^\mu{}_\nu = \begin{pmatrix} 0 & \beta^1 & \beta^2 & \beta^3 \\ \beta^1 & 0 & \theta^3 & -\theta^2 \\ \beta^2 & -\theta^3 & 0 & \theta^1 \\ \beta^3 & \theta^2 & -\theta^1 & 0 \end{pmatrix}.$$

Show that $\omega^{\mu\nu}$ and $\omega_{\mu\nu}$ are antisymmetric.

(e) Show that

$$\theta^i = -\frac{1}{2}\epsilon^{ijk}\omega^{jk}$$

and

$$\beta^i = \omega^{0i}$$

here, i, j, k run from 1 to 3.