Departement Physik, Universität Basel
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Advanced quantum mechanics and quantum field theory, FS 2021

Blatt 2

Submission: 18.03.2021, 12:00H, **on adam in the appropriate folder.**
One file per submission please; the filename HAS TO contain your name, or the submission will not be corrected!
Tutor: Michal Kloc, michal.kloc@unibas.ch

Written test: Friday, 28.05.2021, 13:15 - 15:00
Resources: you may use one **hand**written sheet (2 pages).

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(1) **Relativistic Lagrangian** (3 Punkte)

(a) The relativistic Lagrangian for a free particle of mass m is $L = -mc^2/\gamma$ where $\gamma = (1 - v^2/c^2)^{-1/2}$. Find expressions for the momentum $\mathbf{p}$ and the Hamiltonian H. Discuss the limit $|\mathbf{v}| \ll c$.

(b) Use the Lagrangian $L = -mc^2/\gamma + q\mathbf{A} \cdot \mathbf{v} - qV$ for a free particle of mass m and charge q in an electromagnetic field to derive the Lorentz force, i.e., show

$$\frac{\mathrm{d}}{\mathrm{d}t}(\gamma m\mathbf{v}) = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) .$$

Find expressions for the momentum $\mathbf{p}$ and the Hamiltonian H. Discuss the limit $|\mathbf{v}| \ll c$.

(2) **Lagrangians** (3 Punkte)

(a) Show that the Euler-Lagrange equation for the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

leads to the Klein-Gordon equation. Also derive expressions for the momentum conjugate to the field, $\pi(x) = \partial\mathcal{L}/\partial\dot{\phi}$, and the Hamiltonian density $\mathcal{H} = \pi\dot{\phi} - \mathcal{L}$ and interpret the result.

(b) Find the Euler-Lagrange equations for the following Lagrangian density:
$\mathcal{L} = (\partial_\mu\phi - iqA_\mu\phi)(\partial^\mu\phi^* + iqA^\mu\phi^*) - m^2\phi^*\phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$.

(3) **Adding a divergence to a Lagrangian density** (2 Punkte)
Prove that the equations of motion remain unchanged if the divergence of an arbitrary function of the field $\partial_\nu F^\nu(\phi)$ is added to the Lagrangian density $\mathcal{L}(\phi, \partial_\mu\phi)$.

(4) **Schrödinger equation** (2 Punkte)
The Schrödinger equation can be considered to be the equation of motion of a classical field theory.

(a) Show that the Lagrangian density $\mathcal{L}(\psi, \dot{\psi}, \nabla\psi) = i\hbar\psi^*\dot{\psi} - \frac{\hbar^2}{2m}\nabla\psi^* \cdot \nabla\psi - V(\mathbf{x})|\psi|^2$ leads to the Schrödinger equation.

(b) Calculate the conjugate momentum $\pi(\mathbf{x}, t)$ and the Hamiltonian.