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Theory of Superconductivity, Frühjahrsemester 2020

Blatt 1

Abgabe: 25.2.20, 12:00H (Treppenhaus 4. Stock)
Tutor: Frank Schäfer, Zi.: 4.13

Die **4 Kreditpunkte** und die Note "4" erhält, wer 50% der Punkte aus den Hausaufgaben erreicht. Es wird eine mündliche Prüfung angeboten, in der die Note verbessert werden kann.

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(1) **Meissner effect** (5 Punkte)

The current response of superconductors (at least close to the transition temperature) is described by the London equation,

$$\mathbf{j}(\mathbf{r}) = -\frac{1}{\mu_0 \lambda^2}\mathbf{A}(\mathbf{r}) \,,$$

where $\mathbf{A}(\mathbf{r})$ is the vector potential in the Coulomb gauge ($\boldsymbol{\nabla} \cdot \mathbf{A} = 0$) and λ is a material-specific length. (Further details are described in Tinkham, Sec. 1.2, but they are not relevant for solving the exercise).

Consider a superconducting semi-infinite space ($x > 0$) exposed to an external homogeneous magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_z$ in the z-direction. Calculate and plot the magnetic field and the current density in the superconductor.

(2) **Toy model of Bogoliubov transformation** (5 Punkte)

The effective Hamiltonian

$$H = \epsilon_a a^\dagger a + \epsilon_b b^\dagger b - \Delta b a - \Delta^* a^\dagger b^\dagger \,,$$

contains fermions in two kinds of states a und b (i.e., $\{a, a^\dagger\} = 1$, $\{a, b\} = 0$ etc., here, $\{,\}$ is the anticommutator). We would like to diagonalize this Hamiltonian, i.e., express it in the form

$$H = E_\alpha \alpha^\dagger \alpha + E_\beta \beta^\dagger \beta + E_0$$

by introducing the so-called quasiparticle operators α, β through the following unitary transformation:

$$a^\dagger = u\alpha^\dagger + v\beta \,, \quad b = -v^*\alpha^\dagger + u^*\beta \,.$$

(u, v are complex numbers, α, β are fermionic operators!)

(a) Show that the coefficients have to fulfill $|u|^2 + |v|^2 = 1$.

(b) Express H through α and β, and determine u and v such that H is diagonalized. Determine the energy spectrum of the new quasiparticles, that is, find the expressions for E_α and E_β for the special case $\epsilon_a = \epsilon_b = \epsilon$, and u, v, Δ are real.

(c) Discuss the meaning of E_α, E_β, and E_0.