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Elektrodynamik, Frühjahrsemester 2019

Blatt 9

Abgabe: 7.5.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Michal Kloc Zi.: 4.10
Schriftlicher Test: Donnerstag, 23. Mai 2018, 10.15 - 12 Uhr

(1) **Lorentz-transformed angle** (2 Punkte)
A sailboat is manufactured so that the mast leans at an angle θ' with respect to the deck. An observer standing on a dock sees the boat go by at speed v. What angle does this observer say the mast makes?

(2) **Simultaneity** (3 Punkte)
Event A happens at ($x_A = 5$, $y_A = 3$, $z_A = 0$) and at time $ct_A = 15$; event B occurs at (10,8,0) and $ct_B = 5$, both in system K.

(a) What is the invariant interval between A and B?

(b) Is there an inertial system K' in which they occur simultaneously? If so, find its velocity (magnitude and direction) relative to K.

(c) Is there an inertial system K'' in which they occur at the same point? If so, find its velocity relative to K.

Repeat (a) – (c) for $A = (2, 0, 0)$, $ct_A = 1$; and $B = (5, 0, 0)$, $ct_B = 3$.

(3) **General velocity addition rule** (5 Punkte)
In the lecture we discussed the velocity addition rule by combining two Lorentz transformations. Here, we want to rederive it by using the 4-velocity $v^{\,\alpha} = \gamma(c, \mathbf{u})$ which is a 4-vector. Here, $\mathbf{u}$ is the 3-velocity and $\gamma = (1 - u^2/c^2)^{-1/2}$.
A particle moves with 3-velocity $\mathbf{u}_2$ in K' and K' moves with velocity u_1 along the x-axis of K. Determine the components of the 3-velocity $\mathbf{u}$ of the particle in K and show that $\mathbf{u}$ can be written as

$$\mathbf{u} = \frac{\mathbf{u}_1 + \mathbf{u}_{2\parallel} + \mathbf{u}_{2\perp}\sqrt{1 - u_1^2/c^2}}{1 + \mathbf{u}_1\mathbf{u}_2/c^2}$$

Here, $\mathbf{u}_2 = \mathbf{u_{2\parallel}} + \mathbf{u_{2\perp}}$ where $\mathbf{u_{2\parallel}}$ is parallel to $\mathbf{u}_1$ and $\mathbf{u_{2\perp}}$ perpendicular to $\mathbf{u}_1$. Discuss the cases $\mathbf{u}_1 \parallel \mathbf{u}_2$ and $\mathbf{u}_1 \perp \mathbf{u}_2$.
Hint: Find the 4-velocity $v'^{\,\alpha}$ of the particle in K'. Lorentz-backtransform to K to find the 4-velocity v^α in K. Use the definition of the 4-velocity to calculate $\mathbf{u}$.

(4) **Twin paradox revisited (Griffiths problem 12.16)** (5 Zusatzpunkte)

Problem 12.16 The twin paradox revisited. On their 21st birthday, one twin gets on a moving sidewalk, which carries her out to star X at speed $\frac{4}{5}c$; her twin brother stays home. When the traveling twin gets to star X, she immediately jumps onto the returning moving sidewalk and comes back to earth, again at speed $\frac{4}{5}c$. She arrives on her 39th birthday (as determined by *her* watch).

(a) How old is her twin brother (who stayed at home)?

(b) How far away is star X? (Give your answer in light years.)

Call the outbound sidewalk system $\bar{S}$ and the inbound one $\tilde{S}$ (the earth system is S). All three systems set their master clocks, and choose their origins, so that $x = \bar{x} = \tilde{x} = 0$, $t = \bar{t} = \tilde{t} = 0$ at the moment of departure.

(c) What are the coordinates (x, t) of the jump (from outbound to inbound sidewalk) in S?

(d) What are the coordinates $(\bar{x}, \bar{t})$ of the jump in $\bar{S}$?

(e) What are the coordinates $(\tilde{x}, \tilde{t})$ of the jump in $\tilde{S}$?

(f) If the traveling twin wanted her watch to agree with the clock in $\tilde{S}$, how would she have to reset it immediately after the jump? If she *did* this, what would her watch read when she got home? (This wouldn't change her *age*, of course—she's still 39—it would just make her watch agree with the standard synchronization in $\tilde{S}$.)

(g) If the traveling twin is asked the question, "How old is your brother *right now*?", what is the correct reply (i) just *before* she makes the jump, (ii) just *after* she makes the jump? (Nothing dramatic happens to her brother during the split second between (i) and (ii), of course; what *does* change abruptly is his sister's notion of what "right now, back home" *means*.)

(h) How many earth years does the return trip take? Add this to (ii) from (g) to determine how old *she* expects him to be at their reunion. Compare your answer to (a).