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Elektrodynamik, Frühjahrsemester 2019

Blatt 8

Abgabe: 30.4.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Michal Kloc, Zi.: 4.10
Besprechung: Donnerstag, 2. Mai 2019, 8.15 - 10 Uhr, Hörsaal 1

(1) **Linear antenna** (3 Punkte)
The current distribution of a linear antenna is asssumed to be $\mathbf{j}(\mathbf{x},t) = \mathbf{j}(\mathbf{x})\exp(-i\omega t)$, where

$$\mathbf{j}(\mathbf{x}) = \begin{cases} I\sin(\frac{kd}{2} - k|z|)\delta(x)\delta(y)\mathbf{e}_z, & |z| \le \frac{d}{2}; \\ 0, & |z| > \frac{d}{2}, \end{cases}$$

here, $k = \omega/c$. Calculate the vector potential $\mathbf{A}(\mathbf{r})$ of this antenna in the limit $r \gg d$ without making an assumption about the size of kd. Derive an expression for the time-averaged power radiated per unit solid angle $dP/d\Omega$. Discuss the result, especially in the long-wavelength limit $kd \ll 1$. Compare your result to the dipole result obtained in the lecture.

(2) **Earth vs. pulsar** (5 Punkte)
The magnetic north pole of the earth does not coincide with the geographic north pole – it is off by an angle of about $\beta = 10°$. Relative to the fixed axis of rotation, therefore, the magnetic dipole moment of the earth is changing with time, and the earth must give off magnetic dipole radiation.

(a) Write down explicit expressions for the magnetic and electric fields in the far zone, in terms of the following parameters: β, M (the magnitude of the earth's magnetic moment), and ω (the angular velocity of rotation of the earth). Discuss the polarization of the emitted radiation (linear, circular, elliptical) for an observer that looks at the earth from typical directions.
Hint: write the time-dependent magnetic moment as $\mathrm{Re}\{\mathbf{m}(t)\}$ and use the expressions derived in the lecture.

(b) Write down the formula for the total power radiated in terms of the same parameters.

(c) Using the fact that the earth's magnetic field is about 50 μT at the equator, estimate the magnetic dipole moment M of the earth.

(d) Find the power radiated.

(e) Pulsars are thought to be rotating neutron stars. Which order of magnitude for the total radiated power do you expect for a pulsar with a typical radius of 10 km, a rotational period of 10^{-3} s and a surface magnetic field of 10^{8} T?

(3) **Collinear velocity and acceleration** (2 Punkte)

In the lecture we expressed the angular-resolved power emitted by an accelerated charged particle as

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2}{16\pi^2\epsilon_0}\frac{|\hat{\boldsymbol{\xi}} \times (\mathbf{u} \times \mathbf{a})|^2}{(\hat{\boldsymbol{\xi}} \cdot \mathbf{u})^5}\,.$$

We now assume that the velocity and acceleration are parallel. For fixed acceleration a, make a careful plot of the angular distribution for $\beta = v/c = 0, 0.5, 0.9, 0.99$. Calculate the angle θ_{max} under which the maximal power is emitted. Compare the power emitted in direction θ_{max} for the β's used in the plot and express your result in terms of $\gamma = (1-\beta^2)^{-\frac{1}{2}}$.