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Elektrodynamik, Frühjahrsemester 2019

Blatt 7

Abgabe: 16.4.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Gaomin Tang, Zi: 4.16

(1) **Birefringence** (5 Punkte)

We consider the propagation of plane EM waves in a homogeneous, anisotropic medium with $\mu = \mu_0$ and a dielectric tensor ϵ_{ij} with eigenvalues ϵ_i, $i = x, y, z$. If we choose the principal axes of the tensor ϵ_{ij} as coordinate axes, we have $D_i = \epsilon_i E_i$.

(a) Show that the frequency ω and wave vector $\mathbf{k}$ of a plane wave fulfill

$$\mathbf{k} \times (\mathbf{k} \times \mathbf{E}) + \omega^2 \mu_0 \mathbf{D} = 0 \; . \tag{1}$$

(b) Show that for a given wave vector $\mathbf{k} = k\mathbf{n}$, $\mathbf{n} = (n_x, n_y, n_z)$, $|\mathbf{n}| = 1$, there are two different propagation modes characterized by different phase velocities $v = \omega/k$ that fulfill

$$\sum_{i=x,y,z} \frac{n_i^2}{v^2 - v_i^2} = 0 \tag{2}$$

where $v_i = 1/\sqrt{\epsilon_i \mu_0}$.

(c) We now assume that $\epsilon_x = \epsilon_y = \epsilon_\perp$, $\epsilon_z = \epsilon_\parallel$ (uniaxial medium). Find the velocities of the two propagating modes explicitly for a wave propagating in a direction that has angle θ with respect to the axis of the medium, i.e., $\mathbf{n} = (\sin\theta, 0, \cos\theta)$.
Hint: Reconsider the conditions that you used to derive Eq. (2).

(2) **Radiation fields** (5 Punkte)

The vector potential of a localized (size d) harmonically varying (frequency $\omega = ck$) current distribution at distances $r \gg d$ is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{e^{ikr}}{r} \int \mathbf{j}(\mathbf{x}') e^{-ik\mathbf{n}\mathbf{x}'} \mathrm{d}^3 x' \; .$$

(a) Show that in the far zone $kr \gg 1$, the magnetic field $\mathbf{H} = \dfrac{1}{\mu_0} \boldsymbol{\nabla} \times \mathbf{A}$ and the electric field $\mathbf{E} = \dfrac{i}{k} \sqrt{\dfrac{\mu_0}{\epsilon_0}} \boldsymbol{\nabla} \times \mathbf{H}$ fall off as r^{-1} and are transverse to the radius vector.
Hint: $\mathbf{n} = \mathbf{r}/r$. Use spherical coordinates (r, θ, ϕ).

(b) Compare with the case of a static (time-independent) current distribution.