

Elektrodynamik, Frühjahrssemester 2019

Blatt 6

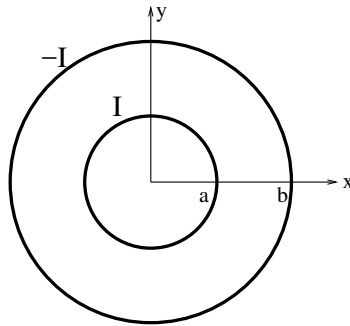
Abgabe: 9.4.19, 12:00H (Treppenhaus 4. Stock)

Tutor: Gaomin Tang, Zi: 4.16

(1) **Coaxial cable** (3 Punkte)

Consider an infinitely long coaxial cable parallel to the z -axis. The inner (hollow) cylinder has radius a and carries the static and homogeneously distributed current I along its axis, the outer (hollow) cylinder has radius b and carries the current $-I$. Both cylinders are uncharged and made of ideal conductors that are assumed to be infinitely thin.

- (a) Calculate the magnetic field $\mathbf{B}(\mathbf{r})$ at $\mathbf{r} = (x, 0, 0)$.
- (b) Calculate the Maxwell stress tensor (in Cartesian coordinates) at the point $\mathbf{r} = (x, 0, 0)$.
- (c) Calculate the force per area acting on the outer cylinder.
Hint: consider a small volume enclosing the point $(b, 0, 0)$.
- (d) Repeat (c) for the inner cylinder.



(2) **Time-dependent current** (4 Punkte)

Assume that an infinitely long straight wire carries the current $I(t)$.

- (a) Give a general expression for the vector potential $\mathbf{A}(\mathbf{x}, t)$.
- (b) Assume that a constant current I_0 is turned on abruptly at $t = 0$. Calculate and discuss the electric field $\mathbf{E}(\mathbf{x}, t)$ and the magnetic field $\mathbf{B}(\mathbf{x}, t)$ in this case. Discuss and plot them as a function of t .
- (c) Calculate the Poynting vector and discuss its magnitude and direction.

(3) **TM modes in a rectangular wave guide** (3 Punkte)

Work out the theory of TM (= transverse magnetic) modes for an infinite rectangular wave guide along the z -direction (dimensions $a < b$ in the x and y -direction).

- (a) Write down the wave equation for E_z and Fourier-transform with respect to z and t .
- (b) What is the boundary condition for E_z ?
- (c) Find the possible solutions for E_z and discuss the relation between frequency and wave vector k_z .
- (d) Plot $\omega(k_z)$ for the two modes with the lowest frequencies.
- (e) Find the wave velocity (= phase velocity) and the group velocities for each mode and discuss your result.