

## Elektrodynamik, Frühjahrssemester 2019

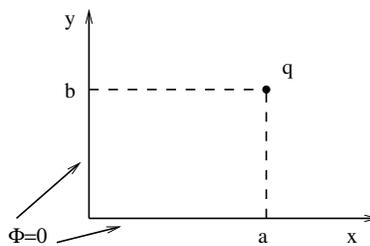
### Blatt 3

Abgabe: 19.3.19, 12:00H (Treppenhaus 4. Stock)

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(1) **Conducting planes at right angles** (4 Punkte)

Two semi-infinite grounded metallic planes meet at right angles. In the region between them, there is a point charge  $q$  situated as shown in the figure.



- Determine the potential  $\phi(\mathbf{x})$ . Hint: introduce image charges such that the potential vanishes on the metallic planes.
- What is the force on  $q$ ?
- Calculate the induced charge density on the metallic planes.
- Suppose the planes meet at some other angle than  $\pi/2$ , say  $\Theta$ ; would you still be able to solve the problem by the method of images? If not, can you find a condition for  $\Theta$  such that the method of images is applicable?
- Repeat (a) for non-metallic plates and Neumann boundary conditions  $\partial\phi/\partial n = 0$ .

(2) **Rectangular metallic pipe** (3 Punkte)

A rectangular pipe, running parallel to the  $z$ -axis (from  $-\infty$  to  $\infty$ ), has three grounded metal sides, at  $y = 0$ ,  $y = a$ , and  $x = 0$ . The fourth side, at  $x = b$ , is maintained at a specified potential  $V(y)$ .

- Develop a general formula for the potential within the pipe.
- Find the potential explicitly, for the case  $V(y) = V_0$  (a constant).
- Evaluate the above potential  $\phi(x, y)$  numerically and plot the result as a function of  $x$  for several fixed values of  $y$ .

(3) **Potential of a charged spherical surface**

(3 Punkte)

Consider a sphere with radius  $R$  that carries the surface charge  $\sigma(\theta, \varphi) = \sigma_0[1 + \cos \theta - \sin^2 \theta]$ , where  $0 \leq \theta \leq \pi$  and  $0 \leq \varphi < 2\pi$ .

Determine and discuss the potential inside and outside of the sphere.

Hint: The general solution of the Laplace equation in spherical coordinates for azimuthally symmetric (i.e.,  $\varphi$ -independent) problems can be expressed as

$$\phi(r, \theta) = \sum_{j=0}^{\infty} (A_j r^j + B_j r^{-(j+1)}) P_j(\cos \theta);$$

Recall that  $P_0(\cos \theta) = 1$ ,  $P_1(\cos \theta) = \cos \theta$ ,  $P_2(\cos \theta) = \frac{1}{2}(3 \cos^2 \theta - 1)$ .

(4) **Induction in a conducting loop**

(just for fun!)

Consider a planar loop containing the resistors  $R_1$ ,  $R_2$ , see Figure. A long solenoid perpendicular to the loop produces a time-dependent magnetic flux  $\Phi(t)$ . What are the voltages measured by the two voltmeters? Discuss the result.

Remark: Assume that the voltmeters are ideal, and that a voltmeter shows  $-\int_a^b \mathbf{E} \cdot d\mathbf{l}$  if  $a$ ,  $b$  are its terminals.

