Departement Physik, Universität Basel
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Elektrodynamik, Frühjahrsemester 2019

Blatt 2

Abgabe: 5.3.19, 12:00H (Treppenhaus 4. Stock)
Tutor: Frank Schäfer, Zi.: 4.13

Termine:
Schriftlicher Test: Donnerstag, 23. Mai 2019, 10.15 - 12 Uhr
Mündliche Prüfungen (Examen): Montag 24. Juni 2019

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(1) **Continuity equation** (2 Punkte)

The continuity equation connects the charge density $\rho(\mathbf{x}, t)$ and the current density $\mathbf{j}(\mathbf{x}, t)$ and reads

$$\boldsymbol{\nabla} \cdot \mathbf{j}(\mathbf{x}, t) + \frac{\partial \rho(\mathbf{x}, t)}{\partial t} = 0 \tag{1}$$

(a) Derive (1) from Maxwell's equations.

(b) Integrate (1) over a volume V bounded by a surface ∂V. Apply Gauss' theorem and explain the physical meaning of each term.

(2) **Sheet current** (3 Punkte)

Assume that there is a homogeneous current in the xy-plane, flowing in the x-direction, with constant current density $\mathbf{j}$.

(a) Calculate the magnetic field $\mathbf{B}(\mathbf{x})$ and a possible form of the vector potential $\mathbf{A}(\mathbf{x})$.

(b) Consider the upper half-space ($z > 0$) and lower half-space ($z < 0$) as two regions and confirm that your result is compatible with the boundary conditions on the fields discussed in Chapter I.3 of the lecture.

(3) **Vector potential and gauge transformation** (2 Punkte)

The vector potential of a long cylindrical solenoid is assumed to have the form

$$\mathbf{A} = \begin{cases} B(-y, 0, 0), & x^2 + y^2 < a^2; \\ \dfrac{B}{2} \dfrac{a^2}{x^2 + y^2}(-y, x, 0) - \dfrac{B}{2}(y, x, 0), & x^2 + y^2 > a^2. \end{cases}$$

Calculate $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$. Find gauge functions $\chi_1(\mathbf{r})$, $\chi_2(\mathbf{r})$ that transform the vector potential in the interior to the forms $\mathbf{A}_1 = \frac{B}{2}(-y, x, 0)$ and $\mathbf{A}_2 = B(-y + x, 0, 0)$, respectively. (Reminder: a gauge transformation changes $\mathbf{A}$ to $\mathbf{A}' = \mathbf{A} + \boldsymbol{\nabla}\chi$).
Which of these vector potentials fulfills the Coulomb gauge condition, $\boldsymbol{\nabla} \cdot \mathbf{A} = 0$?
Sketch $\mathbf{B}$, $\mathbf{A}$, $\mathbf{A}_1$, and $\mathbf{A}_2$.

(4) **Solutions of the Laplace equation** (3 Punkte)

Determine and discuss the most general solution of the Laplace equation $\nabla^2\psi(\mathbf{r}) = 0$ [where $\mathbf{r} = (x, y, z)$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$] with the additional constraint

(a) $\psi(\mathbf{r}) = \psi(r)$, $r = \sqrt{x^2 + y^2 + z^2}$

(b) $\psi(\mathbf{r}) = \psi(s)$, $s = \sqrt{x^2 + y^2}$

(c) $\psi(\mathbf{r}) = \psi(x)$.

Discuss which of these solutions are regular at $\mathbf{x} = 0$ and/or vanish in the limit $|\mathbf{x}| \to \infty$.