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Elektrodynamik, Frühjahrsemester 2019

Blatt 0

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Chapter I of Griffiths provides many more problems and examples in vector analysis.

(1) **div, curl, Gauss, etc.**

Carry out the steps (a) to (c) for the following fields, here, $\vec{r} = (x, y, z)$:
(i) $\vec{A}(\vec{r}) = (y, -x, 0)$, (ii) $\vec{A}(\vec{r}) = (x^2 - y^2, 2xy, 0)$, and (iii) $\vec{A}(\vec{r}) = (x + z, 0, x + z)$.

(a) Calculate $\vec{\nabla} \cdot \vec{A}(\vec{r})$, $\vec{\nabla} \times \vec{A}(\vec{r})$, and $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}(\vec{r}))$.

(b) Calculate the surface integral $\int_S \vec{A} \cdot d\vec{o}$ of the vector field $\vec{A}(\vec{r})$ over the surface S of a cube with edge length 2 that has its geometrical center at the origin (0,0,0).

(c) Sketch $\vec{A}(\vec{r})$. Could we have predicted the result of (b) using Gauss' theorem?

(2) **Electric field of a charged cylinder**

A long cylinder carries a charge density ρ that is proportional to the distance s from the axis: $\rho(s) = ks$ for some constant k.

Find the electric field inside and outside this cylinder.

Hint: Gauss' theorem.

(3) **Stokes' theorem**

Verify Stokes' theorem

$$\oint_{\mathcal{C}} \vec{v} \cdot d\vec{s} = \int_{\mathcal{A}} \vec{\nabla} \times \vec{v} \cdot d\vec{o}$$

for the vector field $\vec{v}(\vec{r}) = (z^2 - y^2, x(z + y), -x(z + y))$; here, $\mathcal{C}$ is the boundary of the disk $\mathcal{A} : \{x^2 + y^2 \leq R^2;\ z = 1\}$.

...and for fun...
(see the end of the course Mechanik)

(4) **Police chases criminals**

As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$. The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$.

Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?

(5) **Communicating with rockets**

A rocket ship leaves earth at a speed of $\frac{3}{5}c$. When a clock on the rocket (system K') says 1 hour has elapsed, the rocket ship sends a light signal back to earth (system K).

(a) According to *earth* clocks, when was the signal sent?

(b) According to *earth* clocks, how long after the rocket left did the signal arrive back on earth?

(c) According to the rocket observer, how long after the rocket left did the signal arrive back on earth?

(d) **(d) and (e) are independent of (a-c)**. Calculate the total energy of the rocket in K and express it in terms of its rest energy.

(e) Calculate the 4-momentum p^α of the rocket both in K and K' and confirm that $p^\alpha p_\alpha = m^2c^2$ is a Lorentz invariant.